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This manual indicates a practical approach to the topic of general or consumer mathematics as taught in senior high school. This course is intended for those pupils who cannot succeed in the sequential high school mathematics course. The material for this course has been selected to provide experiences which will tend to improve the mathematical competence of future workers and citizens. The objectives of these materials are (1) to increase accuracy, understanding, and efficiency in computational skills, (2) to develop new computational skills and extend the understanding of number and computational processes, (3) to provide skill in collecting, reading, organizing, and interpreting data, (4) to develop an attitude of social-mindedness acquired through a study of consumer problems, (5) to provide the mathematical skill and knowledge necessary to cope with the problems of the consumer and citizen, (6) to provide the basic mathematics needed by pupils in their future work and study in the trades and semi-professional occupations, (7) to stimulate an interest in learning mathematics, and (8) to provide an opportunity to demonstrate such traits as creativity, imagination, curiosity, and vizualization. (RP)

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APPLIED MATHEMATICS

STATE OF MINNESOTA
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SECONDARY SCHOOLS
GRADES 10-12

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INTRODUCTION

APPLIED MATHEMATICS

11th and 12th grades

INTRODUCTION

A new and practical approach to the topic of general or consumer mathematics as taught in the senior high school is being attempted in this course of study. To overcome the disinterest prevalent among students taking such a course, caused by the belief that they are repeating the same subject which they did not understand or appreciate in the eighth or ninth grades, an attempt has been made to base the course on student-centered activities of a dynamic and different nature from those that are generally found in such a course. No textbook has been found, to date, with this approach; therefore, a minimum number of basic, readily obtainable references for each topic have been listed upon which the teacher may start building his own file of resource material.

Provision for individual differences is difficult to indicate in an outline. The material is such that a three-tract program in this course could evolve from the subject matter listed. As now organized, the course is directed toward the middle tract.

Function of the Course

This course is intended for those pupils who cannot succeed in the sequential high school mathematics courses. The material has been selected to provide experiences which will tend to improve the mathematical competence of future workers and citizens. By offering this course near the end of their high school education, it is likely that students will be receptive to the topics of the course.

Selection of Pupils

Although there are many items in this outline which would be valuable to all pupils, it is reasonable to assume that it will be "geared" at a lower level than the sequential courses. This course should be reserved for students who take no other mathematics course in grades 10–12. Those in the group who have the ability to complete the sequential courses should be discouraged from enrolling in this course. All other pupils should be advised to enroll for it either in their 11th or 12th year, preferably the latter. Those in the group who rank below desirable standards on a diagnostic test, such as one of those listed below, should be encouraged to enroll in Applied Mathematics.

Within this group of pupils the problem of individual differences is one of major importance; abilities as well as needs and interests will vary greatly. In order to provide for this widely divergent group, a very flexible, rather than a rigidly restricted, course is suggested.

One item of major importance is the use of a competent diagnostic test early in the year to determine the amount and type of remedial work which will be necessary for each individual student. Some suggested diagnostic tests follow:

Basic Skills in Arithmetic Test, by Wrinkle, Sanders, and Kendel. Science Research Associates.

Survey of Arithmetic (Mathematics) *Achievement*, by Tiegs and Clark. California Test Bureau.

Understanding the Meanings in Arithmetic, by David Rappaport. Science Research Associates.

Arithmetic Computation Test, by Madden and Peak. World Book Company.

Organization of the Course

Since this course is to be a flexible outline of material used by the teacher to fit the widely divergent needs of pupils, it is recommended that a constant and thorough review and remedial program be maintained throughout all units to build computational skill. This practice and drill should be informal, frequent, and short, using a variety of activities and experiences, such as games and contests. A helpful booklet, "Games for Learning Mathematics" by Donovan Johnson, can be obtained from J. Weston Walch, Publisher, in Portland, Maine. This review work may frequently be related to the practical problems of the course. The review and remedial work should be tailored to the needs of the individual as well as to the group.

The incentive conditions will have to vary as the motivation of the pupils is sensed. Activities should be student-centered and the teacher must be alert to the rate of learning and the difficulty of the materials. The examples and exercises should be related to the past, present, and future life experiences of the students. For these students the learning experiences should emphasize laboratory work, visualization, and active participation.

The teacher should appeal to the student's imagination and to his background and experience. Let him contribute ideas during the presentation of materials. It becomes important then that as much activity as possible, which is not solely paper-and-pencil activity, be used as part of the classroom procedure.

The students should progress as far as possible within each unit. It will be necessary and feasible to veer from the outline, considering the students' environmental background or lack of previous knowledge. Topics previously covered should be omitted and coordinated with the other departments of the school to avoid unnecessary repetition. The teacher should feel free to add or subsitute other units or topics.

It will be necessary to build a file of resources and activities, drawing upon people in the community for information and sample items. Some helpful free or inexpensive pamphlets as well as some basic references which can be obtained for each member of the class are listed in the suggested aids. There should be much pupil participation and when it is apparent that interest is flagging and motivation difficult, discretion will have to be used as to the advisability of moving on to another topic. Practice in computation should be balanced with the learning of facts and it would be an unusual class which would be interested in all the information listed as subject matter or in doing computational work involving all suggested topics. Many topics may be considered optional, depending upon student ability, background, and interest. For example, the unit on *Fact or Fancy* may be omitted entirely or inserted at any time.

Objectives

1. To increase accuracy, understanding, and efficiency in computational skills.
2. To develop new computational skills and extend the understanding of number and computational processes.
3. To provide skill in collecting, reading, organizing, and interpreting data.
4. To develop an attitude of social-mindedness acquired through a study of consumer problems.
5. To provide the mathematical skill and knowledge necessary to cope with the problems of the consumer and citizen.
6. To provide the basic mathematics needed by pupils in their future work and study in the trades and semi-professional occupations.
7. To stimulate an interest in learning mathematics.
8. To provide an opportunity for the students to demonstrate such traits as creativity, imagination, curiosity, and visualization.

COMPUTING and COMPUTING MACHINES

UNIT I

COMPUTING AND COMPUTING MACHINES

Introduction

Beginning the course with a unit on *Computing and Computing Machines* is an attempt to involve the student immediately in something new and interesting with which to start the school year instead of the traditionally distasteful review of fundamentals. The success of the course will be dependent upon the resourcefulness of the teacher in arousing and fostering this interest and by giving an air of freshness and vitality to the lesson material and classroom procedures.

Essentially the unit, other than giving the student a look at computing aids as a well-established part of our present way of life, serves as a vehicle to review, renew, and establish a certain basic mathematics. The activities for this material should be classroom activities as much as possible. They should not be solely homework activities, or the teacher will be limited to the extent upon which he can capitalize on the results of the activities. Projects should not be time-consuming but accomplished quickly and efficiently while interest remains high.

The effect of this material might be amplified by inviting one or more members of the community who have an acquaintance with computing machines to demonstrate or speak to the class about this equipment.

Objectives

1. To give the students the idea that the course has a fresh approach
2. To give the students a greater appreciation for mathematics as a servant
3. To unite the students into a working entity
4. To review some arithmetic subject areas as a preparation for units to follow and to strengthen the arithmetic backgrounds of the students
5. To teach for an understanding of computing machines as thoughtful inventions

Content	Procedures and Activities	Teaching Aids
A. Motivation for the unit 1. Help the student to see the value of computing machines as time savers and as better insurance toward accuracy of results 2. Help the student see the computing machine as a real part of our present daily life 3. Help the student to feel a desire to participate in the activities of this course unit	Have the students do some time-consuming, error-inducing arithmetic computations, such as finding the sum of a long list of several digit addends, finding the product or quotient of two numbers having six or more digits in their numerals, raising a single digit number to a power greater than ten, for example. Then compare answers for the same problem and discuss the students' reactions to doing these problems Make a class book of pictorial clippings of all or some of the computing machines listed below: adding machines cash registers desk calculators slide rules ammeters voltmeters speedometers numerical tables odometers IBM machines bookkeeping machines electronic computers nomographs	*The Mathematics Teacher*, March 1958. pp. 162-170 The IBM picture series Vorwald, Alan, and Clark, Frank. *From Sand Table to Electronic Brain*. McGraw, 1961 Young, Frederick H. *Digital Computers and Related Mathematics*. Ginn, 1961 In general, most reference titles given elsewhere for this unit where the title alludes to computing machines and devices

Mount several large pockets onto a plain background of some size. Label each pocket with one of the names of the computing machines previously listed. Have the students deposit photographs, clippings, advertising, various distributed materials, and other relevant items into the pockets as they collect them. The teacher may seed some or all of the pockets with materials suitable for instructive purposes and relative to her plans to follow

Have students make a bulletin board of pictures of computers

Any activity used should be meaningful in terms of the unit under consideration and should contribute toward the class members working together as a group

Each activity will have to be organized so that it does not become too time consuming in relation to class time. Projects should be accomplished quickly and efficiently

B. Purposes of computing machines and devices

Invite one or more persons in to discuss their experience with the effectiveness of computing machines and/or devices and with the kind of jobs they will do

Bring in machines that will serve as examples of each purpose listed. Discuss these machines relative to each purpose but do not lecture

Not much time should be spent on this topic

1. To obtain a solution for some problem when the solution would otherwise be unobtainable
2. To obtain a solution which would otherwise be time consuming to obtain
3. To reduce and eliminate errors
4. To avoid the boredom of repeated similar operations necessary to solve certain problems

5. To obtain a concise organized record of the solution for a problem so that any step in the solution may be recovered at a later time		
C. Some computing devices often taken for granted		
1. Names and symbols for numbers	Consider the tediousness of the shepherd counting his sheep by placing them in a one-to-one correspondence with sticks or stones Consider the inconvenience of conveying the idea of a number to the mind of another by always resorting to some sort of visual display such as holding up fingers Point out that symbols for numbers made it possible to record the idea of a certain number for the use of someone not present Symbols for numbers have lessened the demands upon our memories	If much work is done here in numeration systems, it may prove helpful to make a set of trayed strips of tagboard on which the various unit positions are labelled both to the right and the left of a decimal point. Then various numerals may be inserted into the trays to form multi-digit numerals. With a few paper operation symbols and these trayed strips, one can readily display borrowing and carrying, multiplication of numbers, addition of numbers, and so on
2. The digital or place-value system of recording a number with a numeral	Explore how the basic arithmetic operations would have to be carried out when a different symbol is used for each number (requires the use of number line operations). Consider what other demands would be made upon us when a different symbol is used for each number instead of using the digital system Consider in what way giving meaning to the order of digits in a numeral is a convenience What is the value of having a standard base for our numeration system? Note: This is the logical point at which to explore numeration systems with bases other than ten. The extent to which one goes into this topic will depend upon pupil interest, pupil background, available time, for example. Emphasis should be on the meaning of notation, meaning of borrowing and carrying, and on the arithmetic operations. Addition and multiplication tables should	Paper number lines prove useful in some areas here. These aids may be stretched over the front chalkboard and left there during the entire days used to cover the material The Webster Publishing Co. (1154 Redo Avenue, St. Louis, Mo.) has had printed number lines available as part of their advertising circulation in the past. An ingenious teacher can make these lines do for both the positive and negative portions of a number line Films: *How Man Learned to Count*, Associated Films. *A Day Without Numbers*, 10 min Wayne University. *Origin of Mathematics*, 11 min United World Films *Base and Place*. 30 min State U. of Iowa; *Earliest Numbers* 30 min S. U. I.; also

	be used in each system, except the decimal and binary systems. A reasonable coverage might include the systems with bases of five or seven, two and twelve	others of Understanding Numbers series
	A good film on numeration systems will serve well to emphasize the ideas you have been trying to teach and will also serve to tie ideas together and review. If the students have had considerable work on numeration in the past, the film would serve excellently to bring to mind those basic ideas you wish to emphasize as part of the unit at this point	Bakst, Aaron. *Mathematical Puzzles and Pastimes.* Van Nostrand, 1954, ch. 5 Bakst, Aaron. *Mathematics, Its Magic and Mastery.* Van Nostrand, 1952, ch. 1 and 2 Dantzig, Tobias. *Number, the Language of Science. Macmillan, 1954* Asimov, Isaac. *The Realm of Number.* Houghton, 1959 Stein, Edwin I. *Supplementary Units in Contemporary Arithmetic and Elementary Algebra.* Van Nostrand, 1960
3. Memorization of addition and multiplication tables	The majority of your students will undoubtedly know their multiplication tables very well, but many may need to increase their speed of usage by means of flash cards, mental arithmetic, or other games or activities	
4. Arithmetic algorisms a. long division b. long multiplication c. extracting a square root d. finding a greatest common divisor	Review these algorisms as seems useful. If considerable work has been done with operations in the various numeration systems, less work will need to be done here Discuss prime numbers	Some students will have learned different approaches to these operations. The teacher should investigate this situation
5. Extension of one set of numbers to form a larger set a. extension to permit subtraction of all possible pairs (signed numbers and zero) b. extension to the set of rational numbers to permit all possible divisions except by zero c. extension to the set of number pairs to permit the picturing	The extent to which work is done in these areas will again depend upon the needs of the students. A substantial review of adding, multiplying, subtracting, and dividing fractions will usually be beneficial, however If graphing is studied let the students feel that the real purpose of a graph is to pictorially represent relationships and that each type of graph has its own usefulness and limitations. Do not be afraid to step out from traditional types of graphs to get these notions across. Try some of	

of certain relationships among numbers	the following: a. Non-perpendicular axes b. Different sized units on the axes c. Use the second coordinate to determine horizontal distance and the first coordinate to determine the vertical distance. Compare to standard procedure	
	Compare black-and-white graphs to graphs using color to emphasize ideas contained by data. Consider bar graphs, pie graphs, broken line graphs, and other common graph systems	*Road Maps of Industry*. Nat'l. Industrial Conference Board, Inc. 460 Park Avenue New York 22, N. Y. Free
	Compare three dimensional graphs to two dimensional graphs	
	Graph simple inequalities and compare to graphs for equalities	
	Investigate the graphical solution of problems Keep problems simple	
6. Orientation of the decimal system from left to right in its numeration	The results of an opposite orientation could be explored	
D. Short cuts in computing	Emphasis should be given to the fact that each technique is for a specialized situation, and that to make these methods a part of one's tools he must become adept in recognizing each special situation. Considerable drill will be necessary before this will be brought about. These drills should be short, frequent, and cumulative in techniques used	Bakst, Aaron. *Mathematical Puzzles and Pastimes*. Van Nostrand, 1954
		Bakst, Aaron. *Mathematics, Its Magic and Mastery*. Van Nostrand, 1952, pp. 125-133
		Meyers, Lester. *High Speed Math*. Van Nostrand, 1947
		Sticker, Henry. *How to Calculate Quickly*. Dover, 1955
1. Multiplications a. by tens and powers of ten b. by 5, 25, 50, 250, 500, and so on c. by numbers differing from a power of ten or one of the first nine multiples of ten by a difference of one or two	An excellent climax to this material would be a speed contest between opposing teams or individuals. Such a contest should employ some of the more dramatic television approaches Example: $58 \times 72 = (60 - 2)\ (70 + 2)$	Johnson, Donovan A., and Glenn, William H. *Short Cuts in Computing*. Webster, 1961 Film: *Quicker Than You Think*. Associated Films

2. Squaring numbers
3. Addition
   a. by looking for pairs which give easy sums, in the digit columns
   b. addition of 9, 99, 999 and so on
   c. addition of the same number several times
4. Subtraction
   a. of 9, 99, 999
   b. 8, 98, 998
5. Division
   a. tests for divisibility by 2, 3, 4, 5, 8, 9, 10
   b. tests for divisibility by 6, 7, 11, 13
6. Checking of problems
   a. by casting out nines
   b. by use of inverse operation
   c. by approximation (rounding off)

Discuss inverse operations and show their relation to checking

Approximation techniques can be used for estimating answers prior to working a problem and as a crude check for accuracy. Both uses should be developed with the students

E. Some computing devices and simple computing machines

The key here lies in having the students actually make and use some of these devices. The teacher may wish to provide some of the devices to save time, but she should not provide them all. A good deal of learning will take place through their construction. Any ideas a student may have to modify or improve the devices and machines may well be built into his construction

If it is desirable to have each class member have one or more of these devices, most of them can be made inexpensively and simply with a little thought

Perhaps a few students are interested in becoming adept at operating an abacus or soroban and would be willing to put on a display after a little practice. Or maybe there is some person readily available from the community who is

Adler, Irving. *Thinking Machines*. John Day, 1961, ch. vi

Bakst, Aaron. *Mathematical Puzzles and Pastimes*. Van Nostrand, 1954, ch. 13

N.C.T.M., *18th Yearbook*, Columbia Teachers College Press, 1945, pp. 164-181, 117-129, 160-163

Bakst, Aaron. *Mathematics, Its Magic and Mastery*. Van Nostrand, 1952, pp. 113-124, 264-298

Johnson, Donovan A., and Glenn, William H. *Computing Devices*. Webster, 1961

	adept at one of these machines who would be willing to come in	
1. Sliding calibrated lines 2. The abacus and soroban 3. Linkage computers (require the use of ration and proportion) 4. Finger computing 5. Nomographs 6. Napier's bones or rods	Hectograph or mimeograph stencils may be cut for the basic pieces necessary to make sliding calibrated lines Lattice method of multiplication is a good introduction to Napier's bones	
F. Exponential computing	The extent to which one goes into this area will depend upon the maturity of the students involved If the slide rules are used, one should associate them with sliding calibrated lines Logarithmic tables may be used to perform arithmetic operations even though the student does not entirely understand why the system works, but he should be given to understand that he is making use of the properties of exponents	Standard algebra and advanced algebra texts Johnson, Donovan A., and Glenn, William H. *Computing Devices*. Webster, 1961 pp. 15-32
1. Meaning of factor and exponent 2. Multiplying of powers of the same base by addition of exponents and the use of an exponential chart 3. Division of powers of the same base through the subtraction of exponents and the use of the chart 4. Raising a power to a power by multiplication of exponents 5. Simple three-place logarithms using the base of ten, including use of the table 6. Slide rules		
G. Two basic types of computers	Field trips to places using computing machines can be useful here, if the person conducting the tour is prepared to point out such things as the kind of job each machine will do and will emphasize the fact that it is the man operating the machine who must be the brain and not the machine. The following trips are examples of what might be done: 1. Visit a large metropolitan bank	Adler, Irving. *Thinking Machines*. John Day, 1961 ch xi Eckert, W. J., and Jones, R. *Faster, Faster*. McGraw, 1955 Berkeley and Wainwright. *Computers, Their Operations and Applications, Section 1*. Reinhold Pub., 1956

2. Visit an I.B.M. or Burroughs manufacturing plant

3. Visit historical displays of older models of computing machines. These older models do not hide the construction of the machines working parts and can be most instructive

A speed contest using the desk calculator and an electronic brain would be interesting here

Have pictures of each type to show as they are being discussed

Richards, R. K. *Arithmetic Operations in Digital Computers*. Van Nostrand, 1955

Model electronic computers are available

1. Digital computers
   a. definition
   b. examples—adding machines, desk calculators, cash registers, abacus, mechanical bookkeeping machines, telephone dials, many electronic computers

2. Analog computers
   a. definition
   b. examples—logarithmic slide rules, automobile speedometers, electrical meters, odometers, thermometers, clocks, many electronic computers

H. Other ideas involved with computing machines
   1. Punch card selection
   2. Electricity and magnetism as used in designing a modern computer
   3. Finding physical analogies of the binary number system, such as electrical circuits and their switches
   4. Historical order for computing techniques, computing devices, and computing machines

These are topics some students might wish to pursue on their own. Or they may be used to extend the unit if the interest of the class has been high

Adler, Irving. *Thinking Machines*. John Day Co., 1961, ch. x, xii

Bakst, Aaron. *Mathematical Puzzles and Pastimes*. Van Nostrand, 1954, ch. 4

Berkeley and Wainwright. *Computers, Their Operations and Applications*. Reinhold, 1956

Boehm, George W. *The New World of Mathematics*. Dial Press, 1959, ch. 3

The Mathematics Teacher. March, 1958. pp. 169-180

The Mathematics Teacher. March, 1959. pp. 471-478

Reinfeld, N. *Mathematical Programming.* Prentice-Hall, 1958

Culbertson, J. T. *Mathematics and Logic for Digital Devices.* Van Nostrand, 1958.

Johnson, Donovan A., and Glenn, William H. *Computing Devices.* Webster, 1961. pp. 37-51

Newman, James. *The World of Mathematics.* Simon and Schuster, 1956. pp. 2070-2123

PROBABILITY, RISK, and INSURANCE

UNIT II

PROBABILITY, RISK, AND INSURANCE

Introduction

This unit proposes the use of the mathematics of probability to make practical the concepts of insurance. Each student has been or will be faced with the problem of the importance of various types of insurance being sold for protection against 1001 things, including the risks of life itself. The students should not only be made aware of the risks involved in many events of life, but also be made to realize the probability of chance as it affects the success or failure of each event. Many interesting mathematical experiments should be conducted by the students so that they may see first hand the results of chance. All of the experiments with probability must be closely guided by the teacher so as to transmit meaning to the study of various types of insurance. It might be advisable to divide the class into work groups for experimenting with various events concerning probability. These groups would then be responsible for reporting their results to the whole class. Many concepts concerning the collection of data can be taught in this unit that will be helpful in teaching Unit IV. Provide the class with as many teaching aids, activities, and resource people as possible to make the material meaningful.

Objectives

1. To recognize the risks in daily life and learn how to meet these risks
2. To learn how to determine the probability of an event
3. To learn the role of probability in industry, science, government, and particularly in insurance
4. To learn the purpose, types, and benefits of varied insurance contracts
5. To build competence and interest in independent reading of mathematical material and business contracts

Content	Procedures and Activities	Teaching Aids
A. Introduction		
1. The purpose and objectives of this unit	Discuss the objectives of the unit. Use the film *How's Chances* as an introduction	Film: *How's Chances*
	Read section of *How to Take A Chance*. Illustrate the topic with historical incidents	Book: Huff, Darrell. *How to Take a Chance*
		Pamphlet: Johnson, Donovan. *Probability, Risk and Chance*
2. Preview of the topics and activities of the unit	Have students list the risks of the world in which we live: accident, illness, weather, catastrophe, sports, purchases	News and data from newspapers and periodicals
a. The risks in the world in which we live		
b. The determination of risk by experiment	Find examples of the risks in the news of the day	
c. The determination of risk by analysis	Consider what occupations or industries involve chance and what risks or chances are involved: politician, farmer, lawyer, teacher, biologist, weather forcaster, pilot, doctor, for example	
3. Assignments of unit learning activities	Outline the source of information for the unit, the assignments and responsibilities of the students	Bulletin board display on probability or insurance
		Charts: *Family Needs for Life Insurance*

a. Perform experiments and keep a record of the results in a notebook b. Collect articles, news reports, or advertisements related to chance and insurance c. Collect insurance policies for analysis and study d. Make a display, model, exhibit, or game illustrating chance, permutations, mathematical experimentation or insurance	A student notebook for supplementary materials can be very useful A collection of pictures, insurance policies or news material, mounted and labeled can be an appropriate part of this notebook Have a variety of pamphlets and books on probability and insurance in the classroom Make a bulletin board display with objects which can be manipulated to form permutations Have an insurance representative, a highway patrolman, or scientist describe the role of probability or insurance in their work	Speaker: patrolman, insurance representative, or research scientist
B. Probability—The Measure of Chance 1. What is chance? What is the definition of probability? What are equally likely events?	Begin with a discussion of chance Discover a way to measure chance Generalize to the formula $\frac{s}{f+s}$ or number of ways for a successful event to occur divided by the total number of ways the event may occur. Estimate the probability of a variety of events such as tossed coins, dice, or cards If these objects are not appropriate for use in your community use spin dials, dominoes, checkers, or home made cards or even flash cards Make regular polyhedrons such as octogen or dodecahedron or pyramids for use instead of dice	Objects for analysis of events: coins, dice, cards, polyhedra, spin dial, thumb tacks
2. How is the probability of an event by experimentation determined? a. Equally likely events	Have the students determine what the probability should be for each event. This should be emphasized Experiment with the tossing of coins, cards, blocks, dice, checkers. Spin dials, tops, balls Draw marbles, cards, dominoes, letters, for example, from bags Write the probability of the events	Johnson, Donovan. *Probability, Risk, and Chance* Mosteller, Rourke, and Thomas. *Probability and Statistics* Second year algebra textbook with section on probability

	Compare the obtained probability with the estimate of probabilities This should be emphasized to show mathematically how one can closely determine the actual result	School Mathematics Study Group *Mathematics for Junior High School*—Volume II, Part II
b. Events not equally likely	Experiment with events not equally likely such as tossing a thumb tack, tossing a double dice (two dice glued together), odd shaped blocks, spinning a dial, and irregular polyhedrons—guess first what the probability might be	
3. Determine the possible ways different events can occur	Make an analysis of possible events to determine the possible events a. one coin, two coins, three coins b. one dice, one coin and one dice, two dice c. marbles in a bag d. families of one child, two children, three children	
4. The comparison of predicted events and sample results	a. Make predictions of events based on the possible events listed. Compare the predictions with actual results in tossing coins, dice, drawing marbles or cards from a bag b. Compare predictions of weather or sports events with the actual events c. Compare the predictions of events not equally likely such as tack tossing d. Compare the predictions of compound events such as tossing a total of 7 with two dice with obtained results. Compound events can involve any two of the objects of the experiments described above	Check the newspapers for information on predictions and later for the results. Use weekly weather predictions and predictions of football games
C. Probability and Everyday Events 1. The interpretation of probability The law of averages	Compare the results of experiments of individuals in coin tossing with the combined results of all tosses to illustrate the law of averages Find probabilities of events, such as weather, thumb tack tosses, proportion of e's in a paragraph, cards in a deck and compare the predictions with results obtained by small and large numbers	

2. The mortality table and the probability of death	Use the mortality table to predict death at a given age	Mortality table
	Compile death records to compare the table predictions with actual events. Predict the number of class members who are likely to die in 5, 10, or 15 years	Gather this data for at least one month from a major newspaper
3. Accident rates and prediction of accidents	Compare the state accident rates with the predictions for the weekend, month or year	Insurance data or World Almanac
	Predict the number of accidents expected for the class members during teenage driving	Data on accidents and death
	Make a survey of the community to compare actual accident rates with the predicted rate	Contact your local police department
4. Probability and odds	Compare the probability with odds in coin tossing, tack tossing, dice tossing, dial spinners, accident rates, card drawing, births, daily events	Vital statistics of local newspaper
D. Permutations—or the ways an event may occur		
1. The permutations of all members of a set	Use students, letters, books, digits, sports equipment, gifts, game assignments, dress accessories, household furnishings, cards, to form permutations using all members of a set	
Permutations by experiment	Example: Use a football team 1. Determine all the possible ways a starting 11 could be arranged 2. Determine all the possible ways 33 players could be used by keeping them in their right positions	
Permutations computed by formula	Collect data to furnish a pattern for discovering the formula	
The meaning of factorial	Compute factorials and use it to make a table of factorials	
2. The permutations of some members of a set	Use the same materials as in number one to determine the number of permutations for some members of a set	
Permutation by experiment		
Combinations computed by a formula		

3. The combinations of objects	Use the same material as in number one to illustrate the difference between permutations and combinations	
Combinations by experiment		
Combinations computed by a formula	Material and topics on permutations and combinations can be extended to more advanced topics for high ability students. This topic is suitable for a variety of original illustrations and community applications	
E. The Probability of Two or More Events		
1. The probability of two events occurring together or simultaneously (The intersection of two events A *and* B)	Form the lattice for the sum of two dice	Dice, polyhedra, irregular shaped objects
	Find the probability of tossing a given sum	
	Compare this probability with experimental results.	
	Write the possible events for tossing a coin and a die	
	Find the probability of a given combination Compare the computed probability with experimental results	
	Find the probability of events for several coins or other objects such as two dial spinners or two card packs	
2. The probability of one event or another event (The union of two events A *or* B)	Use the events described in number one above to illustrate the union of two events	Double dice formed by gluing two dice together
	Glue two dice together and toss	Form several sets of double dice by gluing different faces together
	Collect data for the totals turned up	
	Compare these results with unattached dice to illustrate dependent and independent events	
	Collect information from science and current news of compound events	
F. Random Sampling and Probability 1. Ways of selecting samples	Experiments in selecting samples	Table of random numbers

Why sampling is needed, e.g; food, crops, rain, T.V. audience, soil	Discuss reasons why small samples are taken. Discuss ways this should be done to get best results	The Minnesota Poll—Minneapolis Tribune
2. The random selection of samples	The comparison of samples results and the population characteristics by coin tossing, card drawing, daily statistics	
3. Random numbers	Sample random numbers and compare the results. Possible activity: Have a large population of numerals in a box. Select samples from the box. Study the small samples. Compare the results of many small samples with the results of the whole population	
4. Random samples and prediction	Survey the students by samples of birthdays, weights, heights, T.V. viewing, outside activities, hobbies, allowances, for example. Count letters or vowels in sample paragraphs and compare results Find news items or advertisements quoting sample results. Study community problems or opinions such as traffic by sampling Find out how public opinion polls select samples	Speaker: Public opinion poll worker
G. Application of Probability		
1. Mathematical expectation— the product of a probability and a measure of monetary value	Determine the mathematical expectation for a variety of events in sports, lotteries, insurance, crops, college education	Crop reports of U.S. Agriculture Department
The formula for mathematical expectation The use of mathematical expectations in industry, lotteries, farming, insurance	If a person is to receive "M" dollars in case a certain event occurs, and if the probability that the event will occur is "p" then the value of his expectation is "Mp" dollars	
2. Uses of probability and sampling in industry	Survey the class to determine expected purchases of gasoline, candy, shoes, other articles	

Inventories, construction, advertising, quality control	Sample a product such as chalk, ball point pens, paper to predict the quality of the product Sample the information and reading habits of the class to determine the effectiveness of advertising, publicity, community information	
3. Uses of probability in science Errors of measurement Distribution of raindrops, leaves, pebbles Seed pollination and crops Population predictions Superstition and random events	Make a series of measurements of a set of digits and tabulate the results to show the range of errors Make distribution of measures of such items as leaves, kernels of corn on a cob, lines in shellfish, sheets of paper in a purchased ream, for example	Objects for sampling and measurement, including corncobs, sea shells, pebbles, for example
4. Uses of probability and samples in politics Public opinion polls Prejudice and decision	Make an opinion poll of class members, teachers, or parents on some school policy or superstition	
5. Probabilities and averages Expected deviations in weights, heigh[illegible]t scores Deviations in reaction time, strength, jumps, heart beat	Compile measures of class members to compare the distributions with population norms If appropriate extend the ideas of mathematical expectation to gambling devices to teach that gambling does not pay. This must be done with extreme caution and proper examples. Community opinion will need to be checked to determine whether these activities are feasible	Police demonstration of gambling devices as used in dispatching patrol cars
H. Life Insurance 1. The purpose of life insurance—meeting the risks of death	The topics and problems of this unit are discussed with exercises in the pamphlet: *The Mathematics of Life Insurance.* This pamphlet could well be the text for this unit Discuss the role of life insurance in terms of meeting the needs of family for support, educa-	Filmstrip: *How Life Insurance Began* *How Life Insurance Operates* Pamphlet: *The Mathematics of Life Insurance* *Life Insurance Fact Book* Chart: *Family Needs for Life Insurance*

	tion, or business establishment. Illustrate with specific family situation	
	Determine the costs involved which should determine the amount of insurance needed	
2. The types of life insurance—term, ordinary life, endowment, paid-up insurance	Illustrate the types of life insurance with policies furnished by students	Life insurance policies
	Discuss the function of term, ordinary life, endowment, or other policies	
3. The mortality table and the cost of insurance	Relate the cost of insurance to mathematical expectation	The mortality table
	Relate insurance rates to changes in life span, accidental deaths and increase in value of insurance company investments	
4. The benefit payments of life insurance cash surrender value paid-up insurance payment at death retirement payments reading the small print of the policy	Use specific insurance policies to compute benefit payments	Filmstrip: *How Life Insurance Policies Work*
	Compare the benefits with premium payments	
	Compare the amount received under each benefit plan	
	Find the exceptions for which no benefit payments are made	
	Discuss the specific documents needed to qualify for benefits	
5. Life insurance as an investment programming life insurance getting the most out of your insurance dollar	Compare insurance benefits with income from investments	Filmstrip: *Planning Family Life Insurance*
	Relate the cost of insurance to the purpose for which insurance is purchased	
	Consider appropriate budget allocations to life insurance	
	The topic of personal budgets and record keeping is discussed in a later unit in more detail	

		Bring a life insurance representative to the class to discuss policies and answer questions	
		Visit the office of a large insurance company	
		Organize a mutual life insurance company for the class	
		Use the mortality table and operation costs as a basis for determining rates	
		Write a sample policy	
		Establish a means of record keeping	
I. Real Property Insurance			
1. The purpose of property insurance		Illustrate the need for property insurance in terms of community events	Speaker: Insurance company representative
		How is insurance on a movie star similar to property insurance?	
		What are examples of the measure of property insurance?	
		Where and under what circumstances is property insurance required?	
2. Statistics and insurance rates		What is the probability of property damage because of fire, earthquakes, flood, tides, storms, lightning, fallout, dust, insects, decay, heat, freezing, plant life?	Film: *Insurance Against Fire Losses*
		What kinds of insurance policies can be obtained? Illustrate with examples	
		For what kind of destruction is it impossible to insure? Use policies to illustrate exclusions	
3. Types and rates of policies reading the small print settling insurance claims (the role of adjustment)		Relate cost to mathematical expectation of loss	Property insurance policies
		Determine what documents, estimates, or statements are required for collection	
		Discuss the role and cost of adjusters, attorneys, agents, court settlement	

	How do companies share risks?	
	How do companies make it impossible to profit from property damage?	
	How can insurance taxes be reduced by property losses?	
4. Programming property insurance	Relate insurance costs to risks and budgets	
	Organize a casualty insurance company for class members	
	Insure against risks, such as loss or breakage of books, pencils, glasses, sports equipment, dining hall dishes	
	Collect data to determine the risks	
	Calculate the mathematical expectation and determine insurance rates	
	Determine basis for proof of loss	
J. Automobile Insurance		
1. The purpose of automobile insurance	Collect a variety of pamphlets, charts, data, and policies to illustrate problems in automobile insurance	Film: *Casualty Insurance* Speaker: Traffic officer
	Use films, insurance representatives, or traffic officers to present the problem to the class	
	Illustrate the purpose of automobile insurance in terms of local accidents	
	Discuss why insurance is mandatory in some states	
	Debate the issue of required versus optional insurance, state versus private insurance	
2. The types of policies liability collision comprehensive medical payment	Use policies to illustrate the various types of insurance	Automobile insurance policies

	Illustrate benefit payments and the basis for settlement of insurance claims	
3. Statistics and insurance rates	Collect data regarding local accident rates	Local accident news and court cases
	Relate accident rates to insurance costs at different age levels and for different types of insurance	Automobile insurance company pamphlets
	Use local news and court decisions to illustrate payments	
4. Programming automobile insurance	Relate local accident awards to the amount of insurance needed	
	Relate cost of insurance to automobile costs of operation and installment payments	
	Compare the rates of different insurance companies and determine the reason for these differences. How are rates related to the location of residence?	
K. Health and Accident Insurance		
1. The importance of health and accident insurances	Have students bring insurance advertisements, policies, or contracts to the class	Collect information about local hospital charges and medical costs
	Discuss the costs of ill-health or accidents	Pamphlet: *Source Book of Health Insurance Data*
	Relate the occurrence of illness and accident to local hospital rates and medical costs	Film: *For Some Must Watch*
2. The types of medical insurance	Use policies or contracts to illustrate the types of insurance	Health and accident insurance policies
hospital and surgical medical income catastrophe	Determine benefits payable	
	Under what conditions will payment be made?	
	When are benefits not paid?	
	What are maximum benefits?	
	What proofs or documents must be furnished with claims?	

3. Statistics and insurance rates	What is the risk of illness or accident?	Collect data about travel insurance at an airport
	What is the mathematical expectation of insurance policies?	Contact several insurance companies for answers to the questions in this section
	Why do insurance rates vary?	
	Who are unable to obtain health or accident insurance?	
	Why is air flight insurance expensive?	
	Collect data to determine insurance premiums paid by employers or business establishments	
	What kind of examinations or inspirations are required by insurance companies?	
4. Selecting the appropriate insurance	Relate insurance costs to standards of living, budgets, and risk	
	It is usually too costly to insure for all risks. How large a savings account with catastrophe insurance could be used as a substitute for health or accident insurance?	
L. Government Insurance		
1. The purpose of social security	The local office of the Social Security Board will supply you with a variety of pamphlets, charts, films, and speakers about the topic	Pamphlets from the Social Security Board
		Films from the Social Security Board
	The teacher will need to use good judgment in the selection of materials, most of which are not mathematical	
	Why do we need government insurance?	
2. The social security tax	Compute the social security tax on different wage payments	
	Compute the minimum and maximum tax payments made by employees	
	Relate these payments to the amount of retirement or health insurance which could be purchased by these amounts from a commercial company	

3. The benefit payments family benefits in case of death cash payments at death retirement benefits unemployment benefits liability benefits medical payments	Have each student determine the possible benefit payments for his family
4. Eligibility requirements	Determine the requirements for eligibility for these benefits Under what conditions are payments made? Who is not eligible for benefits?

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Filmstrips

How Life Insurance Began
How Life Insurance Operates
How Life Insurance Policies Work
Planning Family Life Insurance

Institute of Life Insurance
488 Madison Avenue
New York, New York

Charts

Family Needs for Life Insurance
Four Basic Life Insurance Policies

Free: Institute of Life Insurance

Films

Casualty Insurance. EBF
Life Insurance—What it Means and How it Works. Associated Films
Insurance Against Fire Losses. EBF
Sharing Economic Risk. Coronet
Measure of a Mean. Associated Films

Social Security. Teaching Films Custodians
From Every Mountain Side. Modern Talking Picture Service
How's Chances. Associated Films
For Some Must Watch. Associated Films

Pamphlets

Clifford, Kieffer, and Sobel. *The Mathematics of Life Insurance.*
Educational Division, Institute of Life Insurance, 488 Madison Avenue
New York 22, New York. This is suitable as a text for this unit

Johnson, Donovan. *Probability, Risks, and Chance.*
Webster Publishing Company, 1963. This is a suitable text for the probability portion of the unit

What Life Insurance Means
Institute of Life Insurance, 488 Madison Avenue
New York 22, New York

Blueprint for Tomorrow
Educational Division, Institute of Life Insurance
488 Madison Avenue, New York 22, New York

Money Management: Your Health and Recreation Dollar
Institute of Household Finance Corporation
Prudential Plaza, Chicago 1, Illinois

Life Insurance Fact Book
Institute of Life Insurance, 488 Madison Avenue
New York 22, New York

Source Book of Health
Institute of Life Insurance, 488 Madison Avenue
New York 22, New York

The LANGUAGE of SCIENCE

UNIT III

THE LANGUAGE OF SCIENCE

Introduction

This unit is designed to aid the student in improving his skills and techniques of computation. The method used to attain this goal is the informal use of the concepts of modern algebra. It is developed from the very familiar everyday situations towards the more mathematical applications. Emphasis should be placed on the discovery and understanding of each new idea as it is taught. Many of the topics may be new to you, the teacher. If so, a little homework on your part will prepare you for the task. This is one unit that must be well planned in advance in order to keep the continuity throughout. Much duplication of materials will be necessary. Make sure your problems cover the topics as you have planned. Many of the topics are basic to an elementary course in algebra. Make free use of the recreational problems related to the unit. They will add much to the interest of the unit. Included in the unit is work with signed or directed numbers. This should be developed from the introduction of signed numbers to the mastery of the four processes of arithmetic. The work of the entire unit culminates in the uses of mathematics in scientific formulas. Before this unit is begun, much preparation will be required in writing lesson materials and problems, gathering project ideas, and searching for fresh ideas for each lesson.

Objectives

1. To help the student obtain and maintain skill in computing with understanding, accuracy, and efficiency
2. To further develop the student's abilities and techniques in general problem solving
3. To further develop the student's intuition (shrewd guessing) and estimation
4. To continue or present the method of discovery
5. To help the student, with the use of symbols and mathematical concepts, make generalizations after discovering a possible truth by verifying an intuition by experimentation (inductive method). This should help the student to become more "symbol" minded.

Content	Procedures and Activities	Teaching Aids
A. How we make use of symbols 1. Symbols used in everyday situations	One method of getting this unit started would be to print the name of some student, say "John," on the board and ask the class if that *is* John. Most of the class will agree that what you put on the board is John. Then ask some student to come to the front of the room. When the student arrives at the front, ask him to shake hands with John. It should not take the class very long to discover what you mean, that is, what you put on the board was not John, but a group of four symbols representing the idea of John. Have the class give other examples in everyday situations where we use symbols to convey certain ideas. Some examples which might be brought out are: barber poles, shorthand notation, Morse code, Braille system for the blind, musical notes, shapes and designs of various symbols used in one's religion, referee and umpire designating various situations	*The Growth of Mathematical Ideas, Grades K-12*, Twenty-Fourth Yearbook, N. C.T.M., Washington, D.C., 1959. pp. 327-369

2. Symbols used in mathematics

If the class does not bring up situations involving symbols in mathematics, the teacher can easily insert some ideas which should lead to some simple symbols used: Roman numerals, π, $\perp$, 5, 17, $\frac{1}{2}$, A for area, for example. At this time put a large two (2) and a smaller three (3) on the board and ask the class which is larger, two or three. Typical replies would have some saying two and others three and finally some student will ask, "What do you mean?" This should then be discussed bringing out that 2 is just a symbol representing the idea of two. Therefore, if we want to talk about the symbol, we should have a special name for it. The special name which we give to the symbol representing a number is "numeral"

Develop a lesson around the differences in concept of "numeral" and "number"

B. Open and Closed Sentences
   1. English sentences
      a. true
      b. false
      c. don't know

In order to get the students to understand what we mean by open and closed sentences, the teacher could present to the students on the blackboard or on typed sheets, a group of English sentences which would contain both open and closed sentences. This group should contain three types of sentences: true, false, can't tell. Example: 1) St. Paul is the capital of Minnesota —true. 2) A Russian discovered America—false. 3) He played for the Minnesota Twins—can't tell. Instruct the students to read each sentence carefully and then indicate which sentences are true and which are false. After giving them enough time to complete this group of sentences, go through and discuss each sentence. If someone in the group or the group itself has not discovered that some sentences cannot be classified either true or false, the teacher can direct the discussion by careful questioning, so that the class will recognize this fact. At this time have each student write at least one sentence of each of the three types

A sentence, which is found to be either true or false will be considered a *closed* sentence. If it is impossible to tell whether a sentence is true or

Much of the material involving open and closed sentences could be found in almost any modern elementary algebra book

false, it will be considered an *open* sentence. *In solving open sentences in mathematics, completely avoid "tell em" teaching*

2. Mathematical or Number Sentence
   a. true
   b. false
   c. don't know

The previous material should present an opportunity for the teacher to point out that mathematical sentences can be classified as either open or closed just as well as was done previously with English sentences. It should be emphasized that in mathematics, an expression of the form $3 + 4 = 7$ which is to read "three plus four equals (or is equal to) seven" is called a sentence. The symbol "=" is regarded as the verb of the sentence. Have the students construct several closed (true or false) sentences of mathematics. At this point a set of exercises involving only simple addition, subtraction, multiplication, and division could be given to the student to have him determine whether or not they are true or false

3. Patterns in Number Sentences
   a. Making open sentences true in terms of addition, substraction, multiplication and division
   b. Solving relating operations
      1) addition and subtraction
      2) multiplication and division
   c. The nature of multiplication
      1) in terms of addition
      2) in terms of division

Ask the students if they could think of any method of writing a mathematical sentence so that it would be classified as "open." A sentence of the form $6 + ? = 8$ is called an open sentence. We could also write that same mathematical sentence $6 + \square = 8$. We agree that "$\square$" may be replaced by any numeral (number symbol). These sentences could be read "six plus some number is equal to eight." We have no way of knowing whether or not this is true until we replace the "?" or "$\square$" by some numeral. If we replaced the "$\square$" in $6 + \square = 8$ with various numerals, many sentences can be formed. Ask the class "How many numerals will make $6 + \square = 8$ true?" followed closely by the question, "How many numerals will make $6 + \square = 8$ false?"

It should become apparent that the "$\square$" is simply holding the place for a number. There are many other symbols which may accomplish the same purpose. For example, the sentences $6 + \square = 8$, $6 + \triangle = 8$, $6 + \square = 8$, and $6 + \diamond = 8$ could be used interchangeably. It is important for the

students to understand that these symbols are just holding a place and are to be replaced by numerals to form closed sentences. We call figures like □, △, ▱, and ◇ number frames

It cannot be emphasized too often that there will be times when we will need to use more than one symbol to hold the place for a number in an open sentence. It is agreed in mathematics that when the symbol holding the place for a number is replaced by a numeral, the symbol should be replaced in other places where it occurs in the same sentence with the same numeral. For instance, $\square + \triangle = \triangle + \square$, we may replace the "□" by "2" and the "△" by "7" and have $2 + 7 = 7 + 2$, which is a true sentence

Having students make true sentences out of open sentences and then studying the patterns, should help the student to see the relation of addition and subtraction to each other. In like manner, the relation that multiplication and division are also inverse operations of one another. Examples:

$$\begin{cases} 43 - \triangle = 16 \\ 16 + \triangle = 43 \end{cases} \qquad \begin{cases} 46 \div \square = 2 \\ 2 \times \square = 46 \end{cases}$$

By making use of patterns such as: $2 \times 6 = 6 + 6$
$3 \times 6 = 6 + 6 + 6$ $4 \times 6 = 6 + 6 + 6 + 6$
and related exercises, we can possibly get the students to see the connection between multiplication and addition

4. Order of operations
   a. Addition and subtraction along with multiplication and division

In order to show a need for "order of operations," the teacher could present several mathematical expressions using more than one operation, such as $4 + 5 \times 6$ and have the students evaluate. There might be some disagreement on whether this should equal 34 or 54. In order to avoid conflicts, some agreement must be made. It has been agreed upon in mathematics that we will perform all of the multiplication and division first and after these have been carried out, we perform subtraction and addition. In case we have a series of additions and subtractions,

we perform these operations in order from left to right. The same is true in case a series of multiplications and divisions are involved in the same problem. (Squaring and square rooting should precede multiplying and dividing also.)

b. Multiplication and division along with squaring and square rooting

1) positive, negative, and zero exponents

2) integral and fractional exponents

If the teacher so desires, this might be an opportune time to lead the class into the idea of exponents, which can be thought of as making use of simplifying notations

$5 + 5 + 5 + 5 = 4(5)$
$10 + 10 + 10 = 3(10)$

In the preceding mathematical sentences, the four and three were used to write each sentence in a more concise form

$(5)\ (5)\ (5)\ (5) = (5)^4$
$(10\ (10)\ (10) = (10)^3$

Here the four and three are once more used to simplify the mathematical statement. In sentences (1) and (2), the four and three indicated the number of fives or tens included in the respective products

$10^4 = 10000$	$10^1 = ?$
$10^3 = 1000$	$10^0 = ?$
$10^2 = 100$	$10^{-1} = ?$
$10^{-2} = ?$	$2^1 = ?$
$7^4 = 2401$	$2^0 = ?$
$7^3 = 343$	$2^{-1} = ?$
$7^2 = 49$	$2^{-2} = ?$
$7^1 = ?$	$1^4 = ?$
$7^0 = ?$	$1^3 = ?$
$7^{-1} = ?$	$1^2 = ?$
$7^{-2} = ?$	$1^1 = ?$
$2^4 = 16$	$1^0 = ?$
$2^3 = 8$	$1^{-1} = ?$
$2^2 = 4$	$1^{-2} = ?$

Inductively, this should lead to the idea that $x^1 = x$; $x^0 = 1$ when $x \neq 0$; and $x^{-1} = \frac{1}{x}$; $x^{-2} = \frac{1}{x^2}$ etc.

Simple way to write 6,000,000,000,000,000 is 6×10^{15}

Using 7^3 as an example:

The small numeral three (3), which is above the line, is called an exponent and a numeral containing an exponent, such as 7^3, is an "exponential numeral." The number being multiplied, the 7 of the 7^3, is the "base" of the exponential numeral

A much simpler way of using repeated multiplication is by giving the desired number to a certain power. Thus, 7^3 is "seven to the third power" or more simply, "seven to the third"

This presents the question of how certain multiplication and division problems can be simplified

$16 = (2)\ (2)\ (2)\ (2) = 2^4$
$8 = (2)\ (2)\ (2) = 2^3$
$16/8 = 2$ or 2^1
$32/8 = 4$ or 2^2
$32 = 2^5$ $\qquad 8 = 2^3$
$(16)\ (8) = (2^4)\ (2)^3 = 2^7$

Having the students notice that the sum (if multiplying) or difference (if dividing) of the exponents of the two exponential numerals is equal to the exponent of the answer. This should be extended so as to include answers which will have a base to a negative or zero exponent

This is an additional computational skill that will aid the students. Others were studied in Unit I

c. Use of parentheses

Parentheses, as used to determine order and to help separate, could be interestingly presented by putting several English sentences on the board. Then by inserting punctuation marks, show how the meaning can be completely changed. Several examples which may be used: 1) No price too high! 2) Slow men working. 3) John said the teacher is very intelligent

Whenever a mathematical sentence or expression contains one of the grouping symbols (parentheses, brackets, braces) this becomes a "sig-

nal" saying "do me first." If in the expression, $4 + 6 \times 2$, we wanted to add first instead of the indicated multiplication, we could simply write this expression $(4 + 6) \times 2$

5. Discovering the solution of open sentences
   a. one operation
   b. two operations

Use the discovery method in getting the students to see a general method of solving open sentences. Open sentences of one operation, making use of addition, subtraction, multiplication, and division should be somewhat mastered before beginning open sentences with two operations

6. Missing Digit Problems

When the interest of the class seems to be waning, some type of material which would act as a change of pace could be helpful. Making use of faded documents, number series, cryptograms, magic squares and missing operation problems can be intriguing, stimulating, and challenging

a. Faded document

Examples:
Faded Document—replace the stars with digits to make the statements true

$$\begin{array}{r} 49* \\ +\ **4 \\ \hline 678 \end{array} \qquad \begin{array}{r} 8** \\ -\ *32 \\ \hline 384 \end{array}$$

$$\begin{array}{r} 37 \\ \times\ 2* \\ \hline 2*6 \\ 7* \\ \hline ***6 \end{array} \qquad \begin{array}{r} 1* \\ \hline 37)*** \\ 3* \\ \hline 11* \\ 111 \\ \hline 1 \end{array}$$

b. Series

Number series—complete the number series:
1, 3, 7, 15, 31, **, ***

86, 78, 67, 53, 36, **, **

c. Cryptograms

Cryptograms—solve by substituting a numeral in place of each letter. Each letter represents one numeral

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Gardner, Martin. *Mathematics, Magic and Mystery*. Dover Publications, 1956

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Jones, S. I. *Mathematical Clubs and Recreations*. S. I. Jones Co., 1940

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	SEND (one MORE solution) MONEY SAVE (four possible MORE solutions) MONEY	Merrill, Helen Abbot. *Mathematical Excursions.* Dover Publications, 1957
d. Magic Square	Magic Number Squares—Have students work with the natural numbers in solving magic squares of third and fourth order. After a certain amount of time, some hints or key numbers may be given. The sums of each row, column, and the two diagonals, must be the same in a true magic square	Meyer, Jerome. *Fun with Mathematics* Dover Publications, 1952 Ransom, William P. *One Hundred Mathematical Curiosities.* J. Weston Walch, 1954
e. Missing operation	Missing operation—what number signals or operation will make the following true? 18 ___ 3 ___ 9 = 15 (9 ___ 5) ___ 7 = 28	
7. Equivalent sentences and the structure of arithmetic a. Discovery of the addition, subtraction, multiplication, and division axioms	Provide the students with enough excercises, have them discover a method, other than trial and error, to find the solution set for an open sentence. They should be able to discover the addition, subtraction, multiplication, and division axioms	
b. Discover the "principle of order" (commutativity) for addition and multiplication	In having the students discover the "principle of order" (commutativity) for addition and multiplication, review the idea that $13 - ? = 8$ becomes a closed sentence when the ? is replaced by some numeral. We may express the same idea by using a frame instead of the question mark. The frame may be of different shapes such as □, △, ▱, ◇, ○. Sometimes, we need to use more than one frame in a sentence to stand for different numbers. Different frames within a sentence must be replaced by different numbers, but identical frames within a sentence must be replaced by the same number For example: $\boxed{4} + 9 = 19 - \boxed{5}$ $\triangle + \square = 4 + \triangle$ Have students do a set of problems leading to the idea that addition is commutative, i.e., $a + b = b + a$. Have students try it for subtraction $8 - \square = \square - 8$	

Again by using frame algebra and inductive reasoning, try to get the students to see that multiplication is commutative. Have students try division for commutativity

$10 \div 2 = 2 \div 10$

e. Discover the "principles of grouping" (associativity) for addition and multiplication

In making use of examples and sets of problems, see if the students can generalize or discover that $\triangle + (\square + \diamond) = (\triangle + \square) + \diamond$ and also that $\triangle \bullet (\square \bullet \diamond) = (\triangle \bullet \square) \bullet \diamond$

This is called the principle of grouping for addition and multiplication (associativity). Do you think subtraction and division are associative?

d. Discover the "distributive property" (distribution of multiplication and division over addition and subtraction)

Have students do a set of problems using frame algebra which leads them to the idea that multiplication and division can be distributed over addition and subtraction. Develop the idea that addition and subtraction cannot be distributive over multiplication or division. In other words, $\square (\triangle + \diamond) = \square \bullet \triangle + \square \bullet \diamond$ is always true but $\square + (\triangle \bullet \diamond) = (\square + \triangle)(\square + \diamond)$ is not always true

1) Multiplying a two place numeral by one place numeral

2) Multiplying a two place numeral by two place numeral

At this time it should be profitable to show what we really mean when we multiply sixteen by three

$$\begin{aligned} 3 \times (16) &= 3 \times (10 + 6) \\ &= 3 \times 10 + 3 \times 6 \\ &= 3 \times 10 + 18 \\ &= 3 \times 10 + (10 + 8) \\ &= 3 \times 10 + 1 \times (10) + 1 \times 8 \\ &= (3 + 1)\,10 + 1 \times 8 \\ &= 4 \times 10 + 1 \times 8 \\ &= 40 + 8 \\ &= 48 \end{aligned}$$

In like manner, if the teacher deems it advisable, we can demonstrate what is meant when we multiply a two digit number times a two digit number

C. Changing English to Mathematical Phrases

In both sections C and D, the major stress should be placed upon the relationship between sentences in the English language and in the

language of mathematics. An understanding of this relationship will be of great help to the student in reading scientific material and in the solving of verbal problems

The expression, "five more than a given number" could be written $6 + 5$, if the given number were six. It could be written as $11 + 5$ if the given number were eleven. It could be written $23 + 5$ if the given number were twenty-three. It could also be written $\square + 5$ where the expression $\square + 5$ represents five more than the given number and the $\square$ stands for the given number

Give the class many examples of English phrases and the related mathematical phrases. The examples could be similar to the following: if Mother baked 30 cookies and placed six cookies in each bag, then she must have used 5 bags. $30 \div 6 = 5$

Examples of both closed and open English-mathematical sentences should help the student determine some of the relationships which exist among numbers. The student should then be ready to supply the mathematical phrase or sentence when given an English phrase or sentence

A student should develop some growth in his problem-solving ability if he can answer the following questions pertaining to his problem

1. What do you know to be true in the problem?
2. What is asked for in the problem?
3. Could you write the problem in the form of a number sentence?
4. Can you solve the number sentence?
5. Is the answer to the problem reasonable?

D. Changing Mathematical Phrases to English Phrases

A good deal of thinking takes place when a student is given a number phrase or sentence and then has to write an English phrase or sentence corresponding to it

E. Inequations or inequalities ($\neq, <, >$) and solution sets
   1. Everyday situations
   2. Mathematical situations

Present to the class the symbol "=" and ask the class what it means and give some examples. Then ask them what they think the symbol "$\neq$" means. It should not be very long before some student or the class develops the idea of "not equal." Boys not equal in ability, one city larger than another, for example, should be discussed to show the students that many things are "not equal"

Introduce "$<$" to represent "is less than" and "$>$" to represent "is greater than." Have the students make both true and false sentences using these inequality symbols

F. Directed Numbers and the Number Line

   1. Need for them and where they are used

Discuss some of the games they all have played that require knowledge of signed numbers

Mehl, William G. *Novel Drill Material for Signed Numbers. The Mathematics Teacher*, Volume 52, No. 4, April, 1959. pp. 247-249

Showing a need for signed numbers

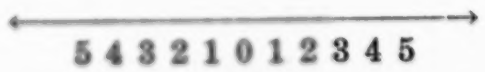
1,	3,	5, ?
2,	4,	6, ?
7,	12,	17, ?
17,	13,	9, ?
25,	15,	5, ?
75,	50,	25, ?
12½,	6½,	½, ?

In order to continue the patterns listed, some type of number other than positive is needed. It is very essential to show a "need" for the negative numbers. A number of situations where the negatives are encountered will help the students to keep uppermost in their minds the "why" which will show results of the "how" later. For example:

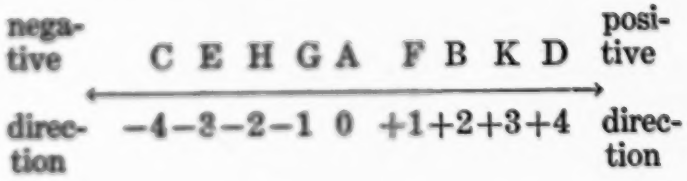
1) overdrawn bank accounts
2) temperatures above and below zero
3) comparing above and below sea level
4) scores of various games
5) gains and losses in football yardage
6) dates of B.C. and A.D.
7) profits and losses
8) north and south latitudes
9) east and west longitude
10) above and below a certain water level predetermined whether or not a flooding condition exists

2. The number line

Introduce by picturing a straight line with arrows on both ends. This indicates that the line continues endlessly in both directions. Choose any point on the line and consider this the starting point or origin. Label this point 0. Mark off equal units to the right and to the left of this 0



We can talk about the two at the right of zero or the two at the left of zero and thus designate which of the two identical numbers we want. We could also designate the left numbers by a symbol such as -5, -4, -3, -2, -1, and the right numbers by $+1$, $+2$, $+3$, $+4$, $+5$. These are called "directed numbers." It is now possible to answer not only "how many," but "which direction." We can now establish a "one-to-one correspondence" to the points on the line and the directed numbers



The symbols $+$ and $-$ now have a dual meaning. The $+$ can mean either addition or direction of a number, while $-$ could mean subtraction or direction of a number

3. Properties of directed numbers

The "property" of opposites or inverses can be demonstrated by listing several pairs of numbers such as:

$$+7, -7 \qquad -8, +8 \qquad -2, +2$$

In comparing left with right, notice each one has an "opposite." Notice that both directed numbers are the same distance from the origin but *opposite* direction. Thus:

$$(+7) + (-7) = 0$$

This leads to the intuitive idea of absolute value. Both $+7$ and -7 are the same distance from the origin, that is, seven units. If we don't care

which direction from the origin but are interested only in the number of units away we use the symbol " $\left|\quad\right|$ " to designate this.

Examples:

$\left|-7\right| = 7 \qquad \left|+5\right| = 5 \qquad \left|-12\right| = 12$

In order to use the intuitive approach in operating with signed numbers, give the students a series of problems such as:

a) Addition

$(+3) + (+2) = ?$ $\qquad$ $(+3) + (-2) = ?$
$(+3) + (+1) = ?$ $\qquad$ $(+3) + (-3) = ?$
$(+3) + (0) = ?$ $\qquad$ $(+3) + (-4) = ?$
$(+3) + (-1) = ?$ $\qquad$ $(+3) + (-5) = ?$

b) Subtraction

$(+6) - (+4) = ?$ $\qquad$ $(+6) - (0) = ?$
$(+6) - (+3) = ?$ $\qquad$ $(+6) - (-1) = ?$
$(+6) - (+2) = ?$ $\qquad$ $(+6) - (-2) = ?$
$(+6) - (+1) = ?$ $\qquad$ $(+6) - (-3) = ?$

This should lead to the generalization that to subtract a signed or directed number, we change the sign and add

c) Multiplication—Geometric Progression

Example: 5, 15, 45, ?
4, −8, 16, ?

4	4	4	4	4	4
×3	×2	×1	×0	×−1	×−2
12	8	4	0	−1	−8

d) Division

$\frac{(-15)}{(-3)}$ = n, means (−3) n = (−15) and that n = +5

$\frac{(\ \ 15)}{(-3)}$ = n, means (−3) n = (+15) and that n = −5

$\frac{(-15)}{(\ \ 3)}$ = n, means (+3) n = (−15) and that n = −5

$\frac{(\ \ 15)}{(\ \ 3)}$ = n, means (+3) n = (+15) and that n = +5

G. Graphing

Graphing is included in some other units. If interested in graphing, please refer to Units I and X

H. Mathematical patterns in science

Patterns, be they numerical, geometrical, or any other type, have interested man through the ages. The ancient Greeks and Egyptians, who were essentially geometers, developed many geometric patterns into their architecture. They also spent considerable time arranging dots into geometric shapes such as triangles and squares, and then enumerating them

Triangle Numbers—

Square Numbers—

Pentagon Numbers—

The Arabs, Hindus, and Greeks seemed to work mostly with number patterns

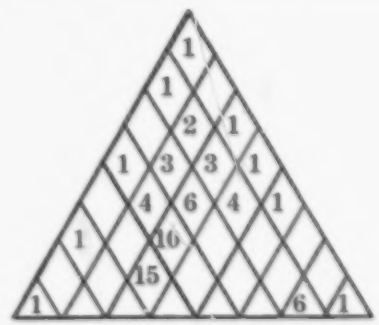
A number pattern or series is the building up of a group of numbers by using some schematic method. Mathematics, being a structure or study of patterns and their relationships, becomes vital and interesting to all of us. Quite often students look at mathematics simply as a course with many rules and a few magic tricks demonstrated by the teacher. The development of new ideas and techniques are often brought about by the studying of patterns, experiments, and their numerical results. It does seem plausible for us to then include in our mathematics

course the study of numerical relationships which display regular and irregular patterns

1. Examining various number patterns

This would be a good time to present to the students many problems. Have them try to complete the number patterns. Some examples which might be used are the following:

a. Look at the triangle and see if you could fill in the blank spaces with the proper number so that the pattern which is indicated is continued



Pascal's Triangle of Numbers (Blaise Pascal —French Mathematician, 1623-1662)

b. The following is also covered in Unit I. Multiply several numbers by 9. Then add up the digits of the products

$9 \times 43 = 387$
$3 + 8 + 7 = ?$
$9 \times 17 = 153$
$1 + 5 + 3 = ?$
$9 \times 400 = 3600$
$3 + 6 + 0 + 0 = ?$
$9 \times 52163 = 469467$
$4 + 6 + 9 + 4 + 6 + 7 = ?$

Can you recognize any pattern which seems to be true in the cases you have examined? If not, try several more

c. From this information, could you tell if the following numbers are divisible by 9 without actually doing the division?
495 6786 245

d. Take each of the numbers from 1 to 10 and square (means times themselves: $5 \times 5, 3 \times 3,$

Glenn, William H., and Johnson, Donovan A. *Invitation to Mathematics*. Exploring Mathematics on Your Own Series. St. Louis. Webster, 1960

Glenn, William H., and Johnson, Donovan A. *Number Patterns*. Exploring Mathematics on Your Own Series. St. Louis. Webster, 1960

4 × 4) them. Compare the number with its square. Do you recognize any pattern?

e. Cutting the Circle
How many pieces do you get if you cut a circle all the way across without cutting through the same point more than twice?

No. of Cuts	Drawing	No. of Pieces	Increase In No. of Pieces

f. Arithmetic and geometric progressions and series problems would fit in nicely here 3, 6, 9, 12 ___ 24, 12, 6, 3, ___ 1, 3, 0, 4, 1, 6, 3, ___

2. Letters stand for numbers—the shorthand for science

Up to this time the students have been using □, △, ◇, and so on. As a placeholder or variable. This would be an opportune time to introduce the idea that we can use any symbol that we want and use it as a placeholder. Sometimes it is convenient to use letters of our alphabet to use as placeholders. These letters should be thought of as numbers and not as letters. Present to the students many of the same problems that contained number frames but now are written in terms of letters

3. Discovering patterns which can be expressed as formulas

It now becomes time for the student to be presented with a problem, with all essential information. The student should then put down this information and look for the pattern. The student's main task would be to express the pattern algebraically. Several problems which could be done in this manner could include:

*a. Time it takes an object to fall to the ground from some height

*b. How far does a car go after the brakes are applied before it stops?

*c. Power in comparison to weight for airplane engines

d. Comparing centigrade and Fahrenheit

e. Ohm's law — Amps = $\frac{\text{Volts}}{\text{Ohms}}$

Sawyer, W. W. *Math Patterns in Science.* American Education Publications. Columbus, Ohio; Wesleyan University, 1960

f. Electric power – Watts = Volts × Amps

g. Sound – Velocity of Sound = Frequency × wave length

4. Discovering simple laws by experimentation

We finally come to the last stage of mathematical patterns in science. The students will now become responsible for setting up an experiment, tabulating the results, looking for a pattern, and then expressing this pattern in terms of a formula. Most of the materials needed to conduct these simple experiments can be found in the science and industrial arts departments or at home. Several examples of experiments which might be conducted in this manner are included below

*a. platform on rockers

*b. pulleys

*c. home-made weighing machine

*d. bouncing of balls, such as, tennis ball, golf ball

e. Hook's law on springs

f. Inclined plane – Mechanical advantage $= \frac{\text{Length}}{\text{Height}}$

g. Clockwise and counterclockwise moment – Force × Distance = Force × Distance

Sawyer, W. W. *Math Patterns in Science*

* Examples may be found in W. W. Sawyer's pamphlet, *Math Patterns in Science*

FACTS from FIGURES and FIGURING from FACTS

UNIT IV

FACTS FROM FIGURES AND FIGURING FROM FACTS

Introduction

One of the purposes of the course is to have the students learn by doing. This will definitely be the key to your success with this unit. The unit must be made practical to the students' interests and abilities. All of the material presented within the unit can be made useful to the student. He must first be shown what statistics is and how it is being used in his everyday life. He must be taught to understand the mathematics of statistics and also to learn ways of analyzing the results. Much practical experience will be gained by formal problems presented by you, the teacher, developed around data gathered from many areas of life. He will also learn much by a project problem designed and studied by himself. Much of the data to be used in the unit could best be collected by the teacher before beginning the unit. This will aid you in the explanation of data collection. One method of presentation that might be desirable would be to teach the unit as presented in the outline and conclude with individual projects by the students which use all the techniques studied in the unit.

Objectives

1. To learn the importance and value of statistics
2. To learn how to collect, tabulate, and graph data
3. To learn how to summarize data by calculating measures of central tendency and rank
4. To learn how to calculate measures of dispersion and to use these calculations in predicting events
5. To learn how to select samples and then use these samples for quality control

Content	Procedures and Activities	Teaching Aids
A. The Role of Statistics in Daily Affairs	Begin by showing examples of statistics found in newspapers, magazines, books, and pamphlets	Huff, Darrell. *How to Lie With Statistics*
1. Why is the analysis of data important? What does statistics involve?	Present a simple set of data, such as test scores for the class. Use the data to illustrate new vocabulary, such as data, statistics, tables, raw scores, distribution. Show the need for organization, arrangement, and summaries of data	Johnson, D. A., and Glenn, W. H. *The World of Statistics*
	Find sources where statistics have been misused. Read these to the class	Wallis, Wilson A., and Roberts, Harry V. *The Nature of Statistics*. Collier, 1962. pa.
	A good bulletin board idea with which to begin the unit might be to work as much vocabulary as possible into a chart or picture diagram of a statistical problem	Census Bureau Reports Bulletin of the U.S. Bureau of Standards Bulletins of the U.S. Department of Agriculture Reports of the U.S. Department of Labor
2. How is statistics used in business, education, industry, science, and government?	What business enterprises are based on statistical data or on the collection of data such as insurance, markets for grain and livestock, business	

	analysts? Discuss government enterprises that collect and analyze statistical data, such as the Census Bureau, the Weather Bureau, the Bureau of Standards, the Department of Agriculture Explain how the data collected is used. For example, get a speaker from the weather bureau to explain the use of statistics in his work	
3. What are sources of statistical data	Go into more detail here as to specific sources for certain types of data. Illustrate the varied types of statistical data found in current newspapers and magazines. What library books or pamphlets are devoted to statistical reports	World Almanac Encyclopedias *Business Week*, *Fortune*, and business periodicals Farm journals, consumer journals
4. What are the personal uses of statistical data?	Discuss community problems which need a solution. Discuss school decisions based on local data, test scores, and accidents, discuss the role of statistics in athletics Have the students keep a notebook in which they collect news, advertisements, articles based on statistics Have students collect original data by surveying students, teachers, parents, regarding activities, opinions, or measures. These measures may be of height, weight, age, shoe size, hat size, heart beat, for example. Data should then be studied later to determine measures of central tendencies and measures of dispersions. A written analysis should be given	Sports data of the local or school paper. Data from the athletic department. Data from insurance companies, weather bureau, other sources
B. Learning to Read and to Present Data	Illustrate by overhead projector or displays or duplications, a variety of tables of data from current reports. Have students collect tables of data and write interpretations of the data. Have each student find a table of data and present it on a sheet of notebook paper. Then write some questions which can be answered by analysis of the table	Data from newspapers, magazines, bulletins, overhead projector and overlays
1. How do we read tables of statistical data?	Stress the importance of writing intelligent questions about the data	
2. How do we construct a table for reporting data?	Use data from a class test, survey, athletic events, or similar local situations to construct a table	

	Discuss the title, column headings, and row labels that will make the data meaningful. Have each student collect data by a survey, an experiment, or by a search of current news and organize this data in a table	
	Develop a series of five to ten sets of defined data which could be used in the development of this unit from short, over-simplified situations, to longer, more involved situations	
3. How do we tabulate scores in a frequency distribution?	Use class data to discuss intervals, interval limits, and data tabulations in a frequency distribution. Apply these principles to a variety of statistical reports to determine where scores are recorded, what are maximum or minimum scores reported	
C. Presenting Data in Pictures		
1. Dot graphs of frequency distributions	Use graphs from current news or Conference Board Reports or almanacs to illustrate types of graphs and how to read them	The Conference Board Weekly graphs
	Have graphs of several types prepared so that students will have examples of each type	Films: *The Language of Graphs.* Statistical Quality Control: Acceptance Sampling Statistical Quality Control: Process Control
2. Bar graphs and histograms	Have students collect at least one example of each type of graph to mount on notebook paper. Write questions for each graph which can be answered by reading the graph	
	Discuss the difference between a bar graph and a histogram	
	Review per cent in terms of finding what per cent a part is of the whole	
3. Circle or rectangular distribution graphs	Have students collect data from the class, school, or community. Draw graphs of this data. Be sure the right type of graph is used and that each graph is properly labeled so that it can be read correctly	
4. Line graphs and compound line graphs	Have students draw different graphs of the same set of data to show how different aspects	

	of the data can be emphasized. Show how graphs change when the scale is changed Also show examples of how to lie with statistics	
5. Frequency polygons		
6. Reading and interpreting graphs correctly	Present by overhead projection, by charts, or by duplication graphs which create misconceptions	Huff. *How to Lie With Statistics* Johnson and Glenn. *The World of Statistics*
D. Measures of Central Tendency and Rank		
1. Mode	Use school and community data for computing measures of central tendency and rank	
2. Medians and rank in a group	Have students find examples of data summaries for their notebook	
3. Mean	Compare the different measures of central tendency for varied distributions	
4. Selecting the proper measure of central tendency for a set of data	Discuss how measures of central tendency or per cents may be used to create false impressions. Stress the importance of accuracy of computation with methods of checking results	Wallis and Roberts. *The Nature of Statistics*
E. Measures of Dispersion	Use local data for computing measures of dispersion	Have a statistician or actuary from a local insurance company or industry talk about statistics in insurance, medicine, and business
1. Range		
2. Average distribution	Use a simplified table of normal dispersion to predict the likely occurrence of a score or an event	
3. Standard deviation	Find examples of data which give distributions approaching a normal distribution	
4. What is the probability of a score or an event?	Use some of the same problems as worked earlier only extending their use for each new concept. Select one or two new sets of data and carry through the analysis	
F. Sampling and Predictions	Review the section on random sampling and probability from Unit II	
1. Why samples are necessary		Table of random numbers

2. How to select random samples	Perform experiments in sampling by drawing cards, marbles, random numbers, counting letters, surveying students, recording daily statistics. Compare the sample results with the characteristics of the entire population. Combine samples to show the results of increasing sample size by sequential sampling	Collections of marbles, cards, pebbles, for example
3. How sampling results are improved by increasing the sample size	Measure average height for the class by a complete census and by sampling and compare results	Quality control statistician from industry
4. How samples are used in industry and government	Have a quality control expert from industry or government discuss statistical methods, sampling and quality control	Public Opinion Poll worker
	A good quality control display is the following: Have one urn with 90 per cent white and 10 per cent black. White is an effective part and black a defective part. This represents the process in control. To represent the machine breakdown, substitute an urn with 50 per cent white and 50 per cent black. How many samples are needed until the breakdown shows up?	Films: *Introduction to Work Sampling*

Data for Investigation

If you want to collect data about current events, the list below will suggest some possible ideas. You can then use these data to apply the principles discussed in this unit. Label completely the data collected, describe the method of collection, draw a graph of the data and write questions that can be answered from the data in your notebooks.

1. Scoring records at athletic events
2. Weather reports—temperature, rainfall, humidity, storms
3. Traffic records—accidents, amount of traffic, number of vehicles, number of parking places
4. School absences or tardinesses
5. School costs
6. Lunchroom or candy sales
7. Vital statistics—births, deaths, marriages, unemployment
8. Recreational activities of friends—radio, movies, books, magazines, sports, hobbies
9. Market quotations—stocks, grains, cattle
10. Newspapers—ads, pictures, comics
11. Business conditions—sales, prices, bank deposits, interest rates
12. Tax rate and expenditures
13. School grades and marks

14. Money in circulation, public debt
15. Electrocardiograms, temperature readings, respiration rates
16. Utility bills—gas, electric, water
17. Use of letters, words, numbers per page in a book
18. Clothes inventory—color, number, type
19. Heights, weights, shoe sizes
20. Distribution of birthdays, and the probability of some birthday falling within a certain time period
21. Coin dates

References

Huff, Darrell. *How to Lie With Statistics*. Norton and Company, 1954

Johnson, D. A., and Glenn, W. H. *The World of Statistics*. Webster Publishing Company, 1961

Mosteller, Rourke, and Thomas. *Probability*. Addison Wesley Publishing Company, 1961

National Conference Board Weekly Graphs. Free. 460 Park Avenue, New York 22, N. Y.

Wallis, W. A., and Roberts, H. V. *The Nature of Statistics*. Collier, 1962. pa.

Films

Introduction to Work Sampling. University of California

The Language of Graphs. Coronet

Statistical Quality Control: Acceptance Sampling. United World Films

Statistical Quality Control: Process Control. United World Films

FACT or FANCY

FACT OR FANCY

UNIT V
FACT OR FANCY

Introduction

Fact or Fancy is a unit on logic designed primarily to give the students an awareness that there can be organization to one's thinking about daily problems divorced from the student's typical emotional involvement. To this end, the examples, situations, and problems involved should be within the life experience of the students and meaningful to them. Lively class participation in all discussions should be encouraged.

As much mathematics as possible can be interspersed in the examples but the teacher should set realistic goals, for there is a divergence of opinion as to just how much logic a high school student is capable of comprehending. A stage may be reached where there are factors of motivation, maturation, and intelligence limits which restrict improvement beyond this point. Symbolic logic, for example, tends to become more and more abstract and a cut-off point may be reached where negative motivation is apparent.

One of the most important objectives in teaching logic is teaching for transfer of learning. Generalizations should be formed and the student taught to recognize the applicability of these rules and principles to situations and problems other than those used in the teaching of these rules. There must be no misconception that logic is being taught for the so-called "training of the mind." By the use of meaningful examples and illustrations the teacher should endeavor to teach the direct value of logic in many fields—social, aesthetic, and utilitarian.

Objectives

1. To learn to apply the principles of logic to mathematics and other specific school subjects
2. To form specific rules of logical reasoning so that generalizations can be formed which can be applied to other situations, especially personal behavioral experiences and the problems involved
3. To learn to recognize fallacies in reasoning
4. To be able to distinguish the difference between truth and validity, deductive and inductive reasoning
5. To study symbolic logic, particularly truth tables
6. To learn about syllogisms and their use in reasoning
7. To learn about the application and use of symbolic logic in computer operations

Content	Procedures and Activities	Teaching Aids
	The teacher can introduce this unit by giving problems whose solution depends upon logical reasoning. For example: the missionary and cannibal problem	Several references will be valuable throughout the entire unit: Johnson, D. A. *Logic and Reasoning in Mathematics*. Webster, 1963 Suppes and Hill. *Mathematical Logic for the Schools*. Stanford Univ. 1961 Standard geometry texts

A. Logical reasoning	Point out the importance of divorcing emotions from logical thinking	Flesch, Rudolph. *The Art of Clear Thinking.* Harper, 1951
1. Characteristics of reasoning	Class should recognize steps: 1. Difficulty is felt 2. Problem is classified and defined 3. Search for clues 4. Suggestions for solution appear 5. Evaluation of suggestions 6. Solution is accepted	
2. Validity	Correct pattern of reasoning which gives valid but not necessarily true conclusions Give examples, such as: All boys are geniuses John is a boy John is a genius	
3. Truth	Basic assumptions true and reasoning is correct Give examples of assumptions or premises from many fields Stress difference between validity and truth	If available, show film: *Do You Know How to Make A Statement of Fact?* Association Films, Inc. Midwestern Branch Office 561 Hillgrove Avenue La Grange, Illinois
	Give details of a murder and have students try to think case through and convince classmates they have reached a logical conclusion	Ripley, H. A. *Minute Mysteries* (Detectograms) Houghton Mifflin, 1932, or comparable material
	Have students mention problems in school life, home life, in relations to members of opposite sex, and discuss steps they would have to take in thinking through to a successful solution. Watch for emotionally-toned analyses	If available, show films: *How to Judge Authorities* *How to Judge Facts* *How to Think* Coronet Films, Inc. Coronet Bldg. Chicago 1, Ill.
	List common steps in these problems and relate to mathematical analysis Students may be led to a comparison such as: *In a mathematical problem:* 1. You must get the whole problem clearly in mind 2. Find out what specific questions are asked 3. Decide what is given in the problem	

	4. Decide what processes or operations of mathematics are suitable to use 5. Estimate the result 6. Perform the computation 7. Check the result *In an everyday life situation:* 1. You must determine what the conditions surrounding the problem are 2. Recognize the real problem. Disregard factors which have no bearing 3. Consider what you know and the assumptions you must make 4. Decide what is the best approach from among the many that might be considered 5. Anticipate the possible solution 6. Arrive at the result you consider to be the best logical conclusion or decision 7. Apply the solution Apply principles discovered to future plans of methods of attack on problems	
B. Definitions and assumptions	Throughout discussion emphasize importance of definition in any argument or discussion	Many geometry texts can provide examples for this section
1. Definitions a. Undefined terms	Give mathematical examples, such as: point, length, surface, space Discuss other examples, such as security, safe driving, happiness	
b. Properties of a definition	By having students attempt to define common terms, such as chair, thermometer, lemon, triangle, for example, lead up to the properties of a good definition, using complete sentences 1. Name the term 2. Place in nearest class 3. Give distinguishing characteristics 4. Use only terms previously defined 5. Test truth of reverse form	

Have students write definitions, such as "ball" as used in baseball to distinguish it from "strike"

Criticize definitions, such as: "Sisters are people who have the same mother"

2. Assumptions

Distinguish from implicative statements which require proof. Intersperse as many examples as possible from mathematics (postulates and axioms)

Give examples in which the student is to give an assumption upon which a statement is probably based, such as:

A baby should not eat hot dogs
To be popular John must buy a car

State assumptions mentioned in the Declaration of Independence

Discuss role of prejudice in acceptance of assumptions we use in life situations, such as racial intolerance

Discuss some assumptions commonly accepted by high school students

Examples:
A car gives one independence
Happiness is dependent upon environment
Once out of school, you'll automatically be happy

C. Inductive reasoning

Characterize steps:

1. Defined terms
2. Premises
3. Generalizations based upon experimentation
4. Conclusion (tentative)

Gives examples of inductive method as a method in mathematics which may establish a high probability rather than a final proof, such as:

$n^2 - n + 41$ is a prime number
(When $n = 41$ it is not true)

Use practical examples, such as:

How inductive reasoning has influenced selection of clothes, progress of air travel, popularity of sayings (Rain before 7, clear before 11), superstitions, for example

D. Deductive reasoning

This topic is an intuitive introduction to syllogisms

Characterize:

1. Valid or invalid argument
2. True or false conclusion

Give examples that can be tested by students for validity and truth, such as:

All squares have right angles

All rectangles have right angles

Therefore, all rectangles are squares

To become a popular student is desirable

Becoming a football player will make you popular

Therefore, becoming a football player is desirable

Flesch, Rudolph. *The Art of Clear Thinking*

Have students reword examples to make the argument valid

E. Syllogisms

Characterize parts of a syllogism:

1. General statement (Major Premise)
2. Particular statement (Minor Premise)
3. Conclusion

Johnson, D. A., and Glenn, W. H. *Sets, Sentences, and Operations.* Webster, 1960

Give examples, such as:

General statement: If equals are added to equals, sums are equal

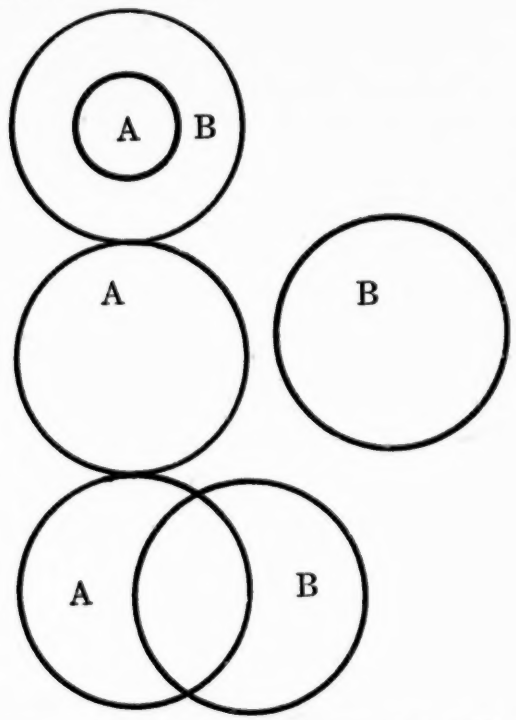
Particular statement: $x - 2 = 6$
Conclusion: $x = 8$

General statement: If I leave school at 3:30, I arrive home at 4:00

	Particular statement: I left school at 3:30 Conclusion: I arrived home at 4:00	
	Have students work examples in which one of the three statements is omitted and must be supplied by them	
	Have students create their own syllogisms to be solved by other members of the class	See "Constructing Logic Puzzles" by Horace Williams in *The Mathematics Teacher*. November, 1961
F. Sets 1. Use in definitions 2. Use in syllogisms	Use visual aids, such as cellophane circles, to clarify Venn diagrams which are models of sets	
	Syllogisms used in previous topic could now be diagrammed	
	Give many examples of use in deductive reasoning. Use both false and true arguments in examples and have students determine truth or falsity by using circles	
	If students have studied sets previously, it may be feasible to go into the topic more thoroughly and involve some Boolean algebra. Keep examples practical and within students' experience. Involve some mathematics in examples	
G. Equivalent transformations (Transformations should be considered optional by the teacher)	To motivate class, consider an exercise in logic such as used by Lewis Carroll: 1. No kitten that loves fish is unteachable 2. No kitten without a tail will play with a gorilla 3. Kittens with whiskers always love fish 4. No teachable kitten has green eyes 5. No kittens have tails unless they have whiskers	Carroll, Lewis. *Mathematical Recreations of Lewis Carroll. Vol. I Symbolic Logic and The Game of Logic*. Dover, 1958 Buchalter, Barbara. "The Logic of Nonsense." *Mathematics Teacher*. May, 1962
	What is the one deduction that can be drawn from this set of statements?	
	Students could be encouraged to bring in examples of this type for a class contest	Wylie, C. R. *101 Puzzles in Thought and Logic*. Dover, 1957

1. Statements of the type "all A is B"
   a. No A is non-B
   b. No non-B is A
   c. All non-B is non-A
2. Statements of the type "No A is B"
   a. No B is A
   b. All B is non-A
   c. All A is non-B
3. Statements containing the word "some"
   a. Some A is B
      Some B is A
   b. Some A is not B
      Some A is B

Use circles to clarify



Examples should be numerous and within the realm of the student's experience. This is the beginning of the use of the tools of symbolic logic and should be an understandable and thought-provoking experience

Have students write equivalent transformations of each of the types

Use multiple choice exercises in which the student has to select equivalent statements

Make a primitive logic machine to show very practical use of transformations

H. Non-equivalent transformations

This topic is included only for completeness of the unit. It is doubtful that there would be many classes interested or mature enough to find it worth while. Omitting this material will not affect understanding of future topics

1. Converse
2. Inverse

Give examples of each type

"Augustus de Morgan and Modern Mathematics." *Mathematics Teacher*. Jan., 1963

Gardner, Martin. "Logic Machines." *Scientific American*. March, 1952

Pfeiffer, John. "Symbolic Logic." *Scientific American*. Dec., 1950

3. Contradictory (negation)		
4. Contrapositive		
a. Indirect proof	Indirect proof could be used to reach a conclusion for such statements as: It did not rain last night The burglar did not enter by this window This figure is not a square	
I. Fallacies in reasoning	This topic could serve, at this point, as a "breather" for those who may have been losing interest in syllogisms. Keep the example and discussion lively	Chase, Stuart. *Guides to Straight Thinking.* Harper, 1956 Huff, Darrell. *How to Lie With Statistics.* Norton, 1954
1. Hidden assumptions (Dicto Simpliciter)	Supply missing assumptions and write arguments in syllogistic form to determine correctness	
	Point out the wide use of unstated assumptions in advertising	
	Students should be able to bring in many examples of practical value in their lives	
	The use of the Latin names of the fallacies may interest someone in making an investigation of the historical background of traditional logic	
2. Irrelevancy (Non Sequitur)		
3. Circular reasoning	A good example of this is the purchase of a car. A job is needed to get support for the car that is needed to get to the job. This could be a heated discussion	
4. Dodging the Issue (Ad Populum)	Point out common uses by students	
5. Hasty generalization		
6. False analogy		
7. Paradoxes	Make a collection from recreational mathematics books	Quine, W. V. "Paradox." *Scientific American.* April, 1962
	Have students analyze articles to see if they can detect the various types of fallacies	de Morgan, A. *Budget of Paradoxes.* Dover, 1954

J. Truth tables
(Optional, but students find this topic fascinating. Sections G and H are prerequisites for this section)

Present a series of statements in which the student, on the basis of a true premise, must decide if the following statements are true, false, or undetermined.

Example:

Premise: It is raining, or it is snowing

A. It is not snowing, and it is not raining
B. If it is snowing, then it is not raining
C. If it is not snowing, then it is raining
D. If it is not raining, then it is snowing
E. It is not the case that it is either raining nor snowing
F. If it is raining, then it is not snowing
G. It is not raining, or it is not snowing

Progress in difficulty to:

Premise: It is impossible for a mallin to have two equal jakes and not be a bulin

A. If a mallin has two equal jakes, then it is a bulin
B. If a mallin is a bulin, then it has two equal jakes
C. If a mallin is not a bulin, then it does not have two equal jakes
D. There exists a mallin which is not a bulin but which has two equal jakes
E. A mallin is a bulin, or it does not have two equal jakes
F. A mallin has two equal jakes, or it is not a bulin

After the arguments ensuing over the correct answers to these exercises students should be ready to appreciate the value of truth tables

See "Developing a Logical Concept in Elementary Mathematics" by Gertrude Hendrix in *The Mathematics Teacher*. December, 1955

1. Terminology

Present in tabular form for future reference

Example:

Spring is a season 3 is a prime number	Prepositions	p q
Spring is a season *and* 3 is a prime number	Conjunction	$p \wedge q$
Spring is a season *or* 3 is a prime number	Disjunction	$p \vee q$
If spring is a season *then* 3 is a prime number	Implication	$p -> q$
Spring is a season *if and* *only if* 3 is a prime number	Equivalence	$p <-> q$
Spring is not a season		$\sim p$

Kattsoff, Louis. "Symbolic Logic and the Structure of Elementary Mathematics" Mathematics Teacher. April, 1962

2. Operations

Practice stating principles of reasoning symbolically and translating symbols into words

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication		
p	q	$p -> q$
T	T	T
T	F	F
F	T	T*
F	F	T*

*Students have difficulty in believing this

Equivalence		
p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Negation	
p	$\sim p$
T	F
F	T

Make truth tables for exercises in Part 1. Make other truth tables but watch for lack of interest

3. Applications	Discuss uses of symbolic logic in digital computer calculations Depending upon abilities, there may be interest in going into a more detailed discussion of the principles underlying the construction of computers and programming	Giles, Richard. "Building an Electrical Device for Use in Teaching Logic." *The Mathematics Teacher*. March, 1962 "Symbolic Logic and Logical Circuitry in the High School." *The Mathematics Teacher*. Jan., 1957 Young, Frederick. *Digital Computers and Related Mathematics*. Ginn, 1961 Johnson, D. A., and Glenn, W. H. *Computing Devices*. Webster, 1961 Gardner, Martin. *Logic Machines and Diagrams*. McGraw, 1958

MAKING a MILLION or SOMETHING LESS

UNIT VI

MAKING A MILLION OR SOMETHING LESS

Introduction

Investments have played an important role in the development of the economy of our country. They are fundamental to developing in the United States the highest standard of living ever enjoyed by a people in all of human history. Investments have helped create small businesses, and have helped those small businesses grow into large corporations. As a result, new products and improved products resulting from industrial research have been developed which in turn make life more comfortable and enjoyable.

The purpose of this unit is to acquaint the student with different types of investments, the place of investments in an individual's savings program, the inner workings of the market, and the interpretation of financial reports.

All investment entails risk. There is no sure way to wealth. Mark Twain's Pudd'nhead once said "April is a particularly risky month in which to speculate in the stock market. The other months are February, December, March, November, May, October, June, August, July, September, and January." It is particularly important that students recognize the risk involved in investments.

Objectives:

1. To develop an understanding of stocks, bonds, and other investments and their meaning and purpose in the economy
2. To develop an understanding of the place of stocks and bonds in an individual's investment program
3. To develop understanding, accuracy, and speed in the computation skills involved

Content

Procedures and Activities

Teaching Aids

The following teaching aids will be useful throughout the entire unit. As there is some duplication in these materials, it should not be necessary to obtain them all. Most of the pamphlets are available free and in quantity.

Make use of Textbooks on Business Mathematics

Films

Behind the Ticker Tape. UW sd color 21 min free

Fair Exchange. Movies USA 21 min free

Opportunity U.S.A. Modern TP 27 min free

Our Shareholders Invest in Tomorrow. Gen Motors 17 min free

Special Report to Stockholders. Gen Mills 7 min free

What Makes Us Tick? Modern TP 12 min free

Work of the Stock Exchange. Coronet 15 min

Working Dollars. Modern TP 13 min free

Pamphlets

Dividends Over the Years. Paine, Webber, Jackson, and Curtis, Pillsbury Bldg., Minneapolis, Minnesota

Understanding the New York Stock Exchange. Paine, Webber, Jackson, and Curtis, Pillsbury Bldg., Minneapolis, Minnesota

How to Read a Financial Report. Merrill Lynch Pierce Fenner and Smith, 240 Rand Tower, Minneapolis 2, Minnesota

New York Stock Exchange Fact Book. Public Relations and Market Development, Publication Division New York Stock Exchange, 11 Wall Street, New York 5, N. Y.

About This Stock and Bond Business. Merrill Lynch Pierce Fenner and Smith, 240 Rand Tower, Minneapolis 2, Minnesota

Dividends for More Than a Decade. Public Relations Department, American Stock Exchange, 86 Trinity Place, New York 6, N. Y.

You and the Investment World, a series of investment folders available by writing New York Stock Exchange, 11 Wall Street, New York 5, N. Y.

I. Investments	There are two techniques which would be quite useful in the introduction to the unit. One approach might be from the point of view of having money to invest, the other being a need for large sums of money for developing a business. The first approach is used here since it is possible to elicit from the students the various types of possible investments. Start by assuming that one or all members of the class have received a large sum of money, say $5,000, as an inheritance from a rich uncle. Assume further that the students have no immediate need nor use for the money. Pose the question: "What would you do with it?" Class discussion should then give you a long list, including most forms of investment. Further discussion should follow in an attempt to answer the question: "What factors should determine where we put the money?" It is now possible to define each form of investment and investigate its characteristics, which is one of the purposes of the unit	
A. Stocks 1. Introduction a. Need for capital b. Purpose of corporation c. Its legal aspects	Discuss need for large sums of money for developing a business, and necessity for legal arrangements to protect owners. Bring newspaper story of successful corporate enterprise, local if possible. Stress possibilities for all economic groups to participate in ownership of corporations	The film *Opportunity U.S.A.* as well as several others listed, would serve well as an introduction. The pamphlet *The American Corporation* in the *You and the Investment World* series would also be good
2. Kinds of stocks, common and preferred a. Dividend differences b. Stability of prices c. Priority of payment	These items are closely related and can be discussed together. Brokerage firms have well-prepared people available to tell the story of stocks to adult classes. Local representatives of these firms may be invited to explain their field to the students. Listed in the teaching aids are related films. Charts are available from brokerage firms and stock exchanges to show comparisons of types of stocks, market trends, and records of mutual funds	Any one of the films listed and most of the pamphlets listed will contribute greatly to this section. In addition, local stock brokers can supply current brochures on typical stocks and mutual funds
	Students should learn how to read a financial page early in the unit. Once common and preferred stocks have been discussed, newspaper quotes of closing prices should be used to acquaint	If newspapers are not available for each student, parts of the financial page can be copied on a transparency and used with the overhead projector

	the students with the standard method of reporting
	Particular emphasis should be placed on the interpretation of the fractions reported for common and preferred stock, the number of shares traded, the Dow-Jones industrial averages and the Standard-Poor index
3. Comparison of common stocks	Investigate the differences and similarities between stocks listed on the New York Exchange, American Exchange, and the Local Market. Speculative and growth stocks are more commonly found in the local market while conservative and cyclical stocks are found on the national markets
a. Conservative	
b. Speculative	
c. Growth	
d. Cyclical	The kinds of stock purchased are usually determined by the amount of risk one is willing to take
e. What are "blue chip" stocks?	
4. Market trends	
a. Bear and bull markets	
b. Correlation with business conditions	
c. History of trends	
5. Mutual investment funds	
a. Diversification	
b. Stability	
c. Professional planning	
d. Rate of commission	
6. Operation of the markets	Take a field trip to the "big board" in Minneapolis or St. Paul or other local stock exchanges
a. Stock exchanges	
b. Brokers' activities	
c. Brokers' fees	

7. Computation skills a. Dividends on preferred and common stocks b. Actual rate of return on investment c. Taxes on dividends and sale of stock	Demonstrate and practice skills involved. This will include the four fundamental operations involving whole numbers, fractions, decimals, and per cents. If possible, obtain income tax forms and practice computing the tax on dividends, and buying and selling profits and losses. Explain dividend exclusions and long term investment benefits in tax computation	Each of the texts listed at the end of this unit has some examples for computation and drill
8. Investment practice	Use the gift of $5,000 and have each student pick his own stock, keep records of profits and losses figured at stated intervals of time. You might choose to limit one group to preferred stock and one to common stocks. Study stocks from: Past record of sale price Present condition and earning power Future prospects Dividend history Debate types of stocks—assume arguments	A daily newspaper with a stock market quotation section will be necessary
B. Bonds		
1. Types a. Mortgage b. Debenture	Present overview of bonds and their security, differentiating between mortgage and debenture. Determine what use is made of bonds in the local community. Compare security and returns to that of stocks	
2. Uses a. Corporation	The purchase of corporate bonds simply means that you are lending money to that organization. You are the creditor, and in return for the loan, the institution pledges to pay you a specified amount of interest on specified dates	
b. United States	The safest bonds in the world. They are payable from the tax revenue of the Federal government. There are many types of government bonds	
c. State	The debts of the states rest on the ability to meet their obligations when due. They cannot	

	be compelled to pay their debts. But states are desirous of maintaing a sound credit rating, therefore default is unlikely. The major advantages are exemption from federal income tax, safety, and liquidity. The rate of interest is low.	
d. Municipal	Major advantage: federal income tax deductions. Disadvantages: low yield, possibility of default	
e. School		A brochure of the local school district pertaining to a recent bond issue will be of interest
3. Computations	Demonstrate and practice skills involved. Note tax exempt feature of certain bonds	
a. Interest		
b. Actual rate on investment		
c. Brokerage fees		
d. Taxes on returns		
C. Speculative Investments		
1. Real Estate	Study and discuss the relationships between population growth, commercial development, and inflationary trends, with the rise and fall of real estate values. Discuss immobility of this type of investment. Local agent may be willing to discuss his work with the class	
2. Contract buying for future delivery	Explain how one can contract to furnish at a future time a certain stock or commodity such as grain, at a set price. Then by playing the market one may be able to purchase the item at a price lower than the delivery price guaranteed, thus realizing a profit. Stress the highly speculative nature of this type of business transaction. A debate might be set up to consider the relative security and possibilities for profit of the various types of investments studied	A local broker would probably have information on this type of transaction, as would the stock exchanges

STRETCHING YOUR DOLLAR

UNIT VII

STRETCHING YOUR DOLLAR

Introduction

In modern society many pressures are exerted upon individuals and families; not the least of these is the competition for the family pay check. Many products designed to make life more satisfying have appeared. It is necessary for students to understand important concepts about money management in order to cope with these pressures. Sound information about the nature of budgets and the costs of installment buying will be an essential element in the life of the student as he plans his future.

Objectives

1. To develop skills related to budgeting at the personal, family, business, and governmental levels
2. To dispel current misconceptions about budgets and their purposes
3. To understand the place of consumer credit in the family's financial plan
4. To recognize the costs of consumer credit buying
5. To understand the advantages and disadvantages of consumer credit buying
6. To understand the difference in cost between credit purchase and the cash price
7. To realize the importance of establishing intelligent buying habits
8. To understand that consumer credit has a place in modern American culture
9. To be able to compute finance charges on installment buying, interest and rate of interest, the true rate of interest
10. To be able to select credit plans which are best suited to the family needs
11. To recognize the need for and ability to establish a good credit rating

Content	Procedures and Activities	Teaching Aids
I. Budgeting	An interesting technique for introducing the unit would be the administration of a money management I.Q. test. The following test was published in a book titled *Consumer Problems and Personal Finance* by Arch W. Troelstrup* This questionnaire, if filled out as accurately as your memory permits, will help you to discover your weaknesses in personal money management. Each "yes" answer rates five points. Add the points to find your money management I.Q. If your score is Over 75, consider yourself a good money manager Between 75 and 55, consider yourself average Between 55 and 35, you are below average Below 35, you are very poor	*Copyright 1957. McGraw-Hill Company, Inc. Used by permission

"1. Have you made a rough plan for your large expenses for the year?

2. Have you kept a written record of your expenditures for at least one month?

3. Have you examined your record of expenditures and made necessary changes?

4. Are you seldom "broke" before your next allowance or income is received?

5. When "broke," do you generally get along as best you can until your allowance is received?

6. Do you avoid making yourself miserable and unhappy by fretting about something you want but cannot afford?

7. Are you in the habit of spending moderately on personal grooming?

8. Can you generally be entertained without spending money?

9. Do you usually resist the spending pressures of friends?

10. Do you resist the spending of money according to your whim without regard to what you really need?

11. When "broke," do you tend to avoid getting an extra sum from your parents or guardian?

12. If you saw a clothing item in a store where you have a charge account, would you be likely to think about how to pay for it before you bought it?

13. Are you careful about not leaving cash in your room or carrying fairly large sums of money on your person?

14. Do you usually avoid buying clothes that you may wear only a few times?

15. Do you spend a moderate amount of money for food between meals?

16. Do you usually save ahead for something you want very much, such as a new dress or suit, a gift, a prom?

17. Do you make it a habit to go to more than one store to compare price and quality before deciding on a big purchase?

18. Would you say that about half your purchases are planned in advanced and are not merely "impulse" buying?

19. Do you know whether your family carries personal belongings insurance, protecting such items as your luggage, clothes, jewelry, golf and tennis equipment?

20. Can you resist buying bargains just because they are advertised as bargains?

A. Common misconceptions

Students and their parents have many misconceptions about budgets. Determine what ideas the students have about the nature and purpose of budgets

Anecdotal records, such as the following can be used to illustrate these misconceptions

1. A budget is *not* bookkeeping

Mrs. James came to the bank with a bookkeeper's ledger containing the year's expenses. "I've kept a record of every penny," she told the budget advisor. Entered in the ledger were such items as "postage, 5c, phone call, 10c," and so on. "And still we can't save money" she exclaimed. Tactfully the expert told her the truth; she had been wasting her time. She had made herself and the family miserable with the mistaken notion that budgeting means keeping a record of everything you spend

2. A budget is *not* a system of fixed percentages

A Chicago housewife told a budget expert that she had calculated everything scientifi-

cally. She knew the formula which said you were supposed to spend "x" per cent on clothes, "y" per cent on housing, and so on. This had called for changing the family's living habits, because they were spending too much on housing. They moved, their rent was lower but they were no longer happy. "Nobody could say how much you 'ought' to spend on housing," the expert told her. There is only what you have to spend and what you want to spend. Buy one thing, and you can't buy another

3. A budget is *not* pinching pennies

Mrs. Smith was on the verge of a nervous breakdown when she came to an adviser. "When we worked out our budget," she said, "figured all kinds of ways to save money. George agreed to cut out two packs of cigarettes per week, Tom, our son, said he could get along on half his allowance, and so on." These things saved the family $5 per week but everyone was unhappy. She had the wrong notion of a budget

B. A budget is a plan for distributing income in order to give every member of the family the utmost satisfaction from the money spent

Students need experiences in establishing budgets. The best place to begin would be setting up a personal budget. This is particularly useful for those pupils who have out of school employment

Ready made account books available from insurance companies, dime stores, bookstores and stationery shops can be used but are not particularly useful for students. They are often too complicated and tend to make the student a bookkeeper

C. The steps in establishing a budget might be generally stated as:

1. Distribution of your known or approximate income
2. Compare expected expenditures with expected income
3. Since wants probably exceed income, the estimates of essential

A budget plan might follow the pattern below:

	Annual Cost	Monthly Cost
Total income	_____	_____
Total expenditures	_____	_____
Net gain or loss	_____	_____
Total budget	_____	_____

Money Management for Your Family. Better Homes and Gardens. Meredith Publishing Co. Des Moines, Ia. 1962

items should be cut, particularly food and clothing

4. Rearrange the rest of the list in order of preference

Taxes		
Federal	_____	_____
State	_____	_____
Social Security	_____	_____
Savings and investments	_____	_____
Total Consumption	_____	_____
Items		
Food	_____	_____
Clothing	_____	_____
Housing	_____	_____
Furnishings	_____	_____
Other Insurance	_____	_____
Advancement	_____	_____
Charity		
Education		
Church		
Recreation		
Automobile	_____	_____
Periodic Expenses	_____	_____
Other items	_____	_____
Total Expenditures	_____	_____

Since every student does not have income with which a personal budget can be established, hypothetical situations can be established using a projected income figure from his future vocational choice

As item costs are estimated, an interesting activity is the comparison of prices of present-day purchases with those a century ago. At that time, the following costs are typical:

Milk 5¢ per qt.
Raisins 7¢ per lb.
Butter 12¢ per lb.
Beef 9¢ per lb.
Cheese 9¢ per lb.
Coffee 25¢ per lb.
Boots $5 per pair
Coats $12
Vest $6

	It would also be advisable to compare salaries as well. Some typical salaries were: Laborer 37½¢ per day Minister $700 per year Farm laborer $ 3 per month, plus board For those students who have difficulty knowing where their money is going, keeping cash accounts can be useful in obtaining some idea as to what items to budget for. It should be clear that the cash accounts should *not* determine the budget. The budget should be established by the amount of income and what the individual wants to buy with it	
D. Family budgets Two cannot live as cheaply as one The most common cause of marital discord is related directly or indirectly to money management	The family budgets the items to be included will differ from the personal budget. The amounts budgeted will also differ Students might gain a good deal of understanding regarding money management by constructing family budgets for their own family. If marriage is in the plans of the pupils, a projected family budget can be devised. It is important to bring out the fact that family budgets should be established only by joint planning of the husband and wife	
E. Business budgets	Explore the problems of budgeting in small businesses, corporations, churches, PTA's or school groups in which the students hold membership The items for these budgets are different but the basic problems remain the same. Sample budgets for some of these groups should be duplicated and handed to students for discussion. Perhaps copies of the school budget are available.	U. S. Bureau of Internal Revenue, Washington, D. C. *Tax Kits* Invite superintendent to talk on school budgets
F. Governmental budgets	Although more removed from the daily life of the individual, such a study may be valuable if not covered in the social studies class	Write your congressman for information. Ask city officials for information
II. Installment buying A. Introduction	This unit should make use of the items listed in	An ideal, free booklet which can be used

This unit should provide the answers to key questions, such as:

1. Where can we go to borrow money?
2. Which institutions are best?
3. What is meant by "true" interest?
4. When should we borrow money to pay for durables instead of buying on the installment plan?
5. How has credit helped expand our economy to the highest standard of living ever enjoyed by a nation?
6. Why is it important for a person to have a good credit rating?
7. What are the advantages and the disadvantages of installment buying?
8. How does a person finance the purchase of an automobile?

the students' budget which cannot be purchased for lack of ready cash. Some students may have mentioned this possibility in their personal or family budgets in the previous section

The remaining part of the outline follows closely the chapters in the pamphlet listed to the right

Have the students list some of the items in their own home which were purchased on the installment plan

Have the students discuss the necessity of obtaining a mortgage when purchasing a home or business

as a text for this section is "Using Instalment Credit," by Clyde W. Phelps, published by Commercial Credit Co., Educational Division, Baltimore, Md.

B. When does one use cash, charge accounts, or installment buying?	Answers to this question can be elicited from students after noting the kinds of items which are purchased by these methods	Chapter I, "Using Instalment Credit"
C. Financing installment purchases 1. Financing institutions a. Sales Finance Co. b. Banks c. Small loan company d. Loan sharks e. Saving and loan ass'n. f. Credit unions	Have the students investigate the characteristics of each type of agency and contrast their operation. A brief study of these many agencies leads directly into a discussion of how one chooses a financing institution and thus to installment charges	Chapter II, "Using Instalment Credit"
D. Installment Charges 1. Necessity of charges a. interest b. bad debt and credit losses c. administrative costs	Have the students play the role of a loan agency and ask them to explore what costs they will have which must be included in the cost to the borrower	Chapter III, "Using Instalment Credit"

2. The dollar cost
On an installment purchase, the dollar cost is the difference between the cash price and the "time price."

Provide the students with several problems, somewhat involved, where the dollar cost is to be found

3. The annual interest rate on purchases. By formula the rate is:

$$R = \frac{2\,m\,I}{P(n+1)} \text{ where}$$

R is the annual cost rate
m is the number of installment payments per year
I is the total amount of the financing charge
P is the *net* amount of the credit advance
n is the number of installment payments called for in the contract

An excellent discussion of this formula is found in "Using Instalment Credit." The formula should not be taught but should be developed slowly as outlined in the pamphlet. Numerous problems can be devised by the teacher where dollar costs and annual interest rates are computed. It is important for the students to realize that both dollar cost and annual interest rate are important pieces of information in making decisions regarding installment purchases

Have the students bring to class some advertisements from newspapers and magazines and try to secure enough information from them to compute dollar cost and interest rate

E. Misconceptions dealing with installment percentage rates

F. Comparing the costs of installment purchases
1. rates
2. services

G. Possible benefits of installment purchases

H. Problems of excessive debt

I. Maintenance of a good credit rating

J. The role of installment purchases in the nation's economy

These topics could probably be handled best by small group discussion periods where the students have an opportunity to develop the concepts on their own. Every effort should be made to keep the unit active and alive, and to provide the students with much activity

Have a student group study about and/or visit a credit bureau and report to the class

Chapter IV, "Using Instalment Credit"

"Using Instalment Credit," Chapters V, VI, IX, X

Films
Banks and Credit. Coronet 10 min. b&w color

Pamphlets:
Using Instalment Credit, Clyde Phelps. Consumer Credit Co. Baltimore, Md.

Money Management Booklets. Household Finance Corp. Prudential Plaza, Chicago, Ill.

Consumer Credit Facts for You. Bureau of Business Research. Western Reserve University, Cleveland 6, O.

FINANCING FREEDOM

UNIT VIII
FINANCING FREEDOM

Introduction

Walk up and down Main Street, attend a county fair, meet with a group of housewives, or visit with workers in industry. One topic of conversation you will undoubtedly hear will be taxes. Almost everyone will be complaining of the amount of money appropriated from pay checks by government—local, state, and federal. Much of this is pure "griping," based upon observation and incomplete consideration of facts. Few people will suggest cutting out specific areas of governmental activities which these taxes finance.

The main spirit of this unit should be one of information. The students should become familiar with the purposes of taxes they pay and will pay. They should be taught in a manner that will foster a truly critical evaluation of the entire tax structure. Mathematical by-products of this study will be numerous. For example, percentage computation and the making of graphs are integral parts of the study of taxation.

Before starting this unit, it would be wise to discuss it with the teachers of social studies and business education in your school. This will point out areas of concentration and avoid unnecessary duplication as well as contradiction.

The usual periods of drill should be continued. Be sure to keep these short, frequent and informal. Again it must be emphasized that this is a *guide* to activities, a set of selected suggested things to do. It is not intended to be followed rigorously. You may elaborate on some sections and omit others. You are urged to examine carefully the column headed "Teaching Aids" well before starting the unit so that helpful materials can be obtained for class use at the proper time.

Objectives

1. To learn the importance of taxes in the life of all individuals
2. To acquire skills which will enable them to fill out their annual income tax returns correctly and accurately
3. To understand why taxes are necessary to maintain the American way of life
4. To better understand the tax structure in the United States

Content	Procedures and Activities	Teaching Aids
A. Taxes		
1. Definitions	Find definitions. Write one acceptable to class. Have a brief report on history of taxation. Discuss "Taxation as a badge of freedom"	Consult *World Almanac* for current year for statistics
2. History		
3. Major classifications		U. S. Bureau of Internal Revenue, Washington, D. C. *Tax Kits*
a. income, private, and corporate		
b. excise or special sales tax		
c. property taxes, real, and personal		
d. general sales tax		
4. Meaning of direct and indirect taxes	Consider some products such as an automobile and investigate indirect taxes	
B. Services paid for by money received from taxation	Ask students to list as many services as they can. This should probably be done in class and	

	put on the chalkboard. Students have strange ideas about who foots the bills	
	Spend a lot of time on this section to lay a good foundation	
C. Governments to which we pay taxes 1. Federal 2. State 3. County 4. Local	Have student list the services under the government which they think pays for these services. Some will be under more than one	
D. Federal Taxation 1. Major sources of revenue	Make a circle graph showing source in per cent	
	This might make a good class project	
2. Major expenditures of the federal government	This might be an individual or committee project. Make a circle graph showing expenditure in per cents. Find what the annual budget is for the present year. What is the average cost per person? How do income taxes in the U. S. compare with those in other countries? What is the present national debt?	
3. Federal taxes you will be sure to pay	Set up a hypothetical family income, source of income	U. S. Bureau of Internal Revenue, Washington, D. C. *Tax Kits*
	Use problems in tax kit. List pros and cons of withholding	
a. Income	Fill out a tax form	
	Explain carefully short and long form	
b. Excise	Make a list of excise taxes paid by the student's family	
E. State Taxes 1. Sources of income	Write for information. Make a circle graph showing percentage	"Report to Governor and Legislature," Commissioner of Taxation, St. Paul
2. Major expenditures of the state	List in order of cost the major expenditures	Research Division, Minnesota Taxpayers Ass'n., 812 Minnesota Bldg., St. Paul
	Make a bar graph of these expenditures	
	Compare state graphs with federal graphs	County Assessor or County Supervisor

3. State taxes the student will be likely to pay a. income b. special sales	Fill in State Income Tax Form for a hypothetical individual and/or family	State Tax Forms
	Find differences between state and federal forms	
	Get suggestions for making tax collections more efficient	
	Find gasoline tax—state and federal	
	What per cent of total cost is the gas tax?	
	What per cent of total cost of cigarettes is the tax?	
	What per cent of total property tax in the community goes to the state?	
	What per cent of states have a sales tax? Panel discussion or debate—"How much would sales taxes be on various amounts?" "What are arguments for and against a sales tax?"	League of Women Voters, 84 So. Sixth St., Minneapolis 14
	It has been said that Minnesota has an unfavorable tax climate for bringing in new industry. How does Minnesota rank as a taxpaying state?	
	How does Minnesota rank among the states in taxes compared to income?	
	Can you find anything concerning corporation taxes?	
F. Local Taxes 1. Major source a. property tax (1) real estate (2) personal property b. licenses 2. Major Expenditures a. schools b. roads c. public service (1) hospital (2) library	Have the county assessor or supervisor come to the school. These questions may be asked: What is meant by mill rate? How is local mill rate determined? How are assessors chosen? What training is necessary? How is full and true value determined? Is this valuation consistent throughout the state? What is meant by homestead and non-homestead land?	County assessors have much information on property taxes

d. protection (1) police (2) fire e. welfare f. recreation	What is meant by agricultural land? How does it differ in the manner of assignment? Who pays personal property taxes? What per cent of the property tax is personal property tax? Some people consider this an unfair tax—why? How could money be raised in its place? Work problems—given market value, full and true value and local mill rate. Given tax and mill rate, what is the assessed valuation?	
G. County Taxes 1. Major source a. property 2. Major expenses	List major expenses List the facts that have impressed you most on Minnesota taxes If you feel taxes are too high, what services would you reduce? Are there any services that could be provided more cheaply by private industry than by local, county, or state government? Get a property tax form from the county assessor or county auditor Find what per cent of property taxes go to the city, the county, the state, and the schools Make a graph	
H. Financing Schools	How much tax money is spent for education? (Federal, state, and local) From your school superintendent find the cost per pupil How is the cost divided between state and local government? Does the school receive any federal aid? For what purposes is federal aid received?	National Education Association has much information on this topic Films *Citizen Dave Douglas* 27 min b & w Modern Talking Pictures Service, Inc., 3 E. 54th St., New York 22, free loan

Make graphs of sources of school aids

Federal Taxation. Coronet 10 min b & w 1948 color

Helping the Taxpayer—What Happens When the Government Questions Your Tax Return 1955 14½ min sd b & w, Association Films, Inc., 347 Madison Ave., New York 17, free loan

Property Taxation. Encyclopaedia Britannica Films

Senator's Daughter 1957 30 min sd color Jam Handy Organization, 2821 E. Grand Blvd., Detroit 11, free loan

Tax Policy—What About it? 1954 15 min sd b & w, 516 Fifth Avenue, New York 36 Am Film Forum Ser.

MONEY and YOUR MANOR

UNIT IX
MONEY AND YOUR MANOR

Introduction

Money and Your Manor is a unit designed to give the student a better understanding of the role of mathematics in the lives of the home owner and home builder. As an integrating activity, each student could be concerned with designing his own home—from the selection of the lot to the furnishing of the home. To involve and interest each student immediately, at the beginning of the unit students could volunteer to serve as contractors, painters, or supply dealers. They could form and name their own companies to supply materials and work on each other's houses.

As the unit progresses the opportunities for computational practices, mensuration problems, approximate number calculations and other applied problems are extensive. To keep the class enthusiasm high it may be necessary to curtail the extent of such practice if a saturation point is felt to have been reached.

There are many more activities listed than would be applicable to the average class and the teacher should be alert to and capitalize upon specific areas of interest due to the community or home environment.

Objectives

1. To learn to appreciate the role of mathematics as applied to the building trades
2. To develop the skill necessary to make and use the measurements required in home construction and maintenance
3. To learn to appreciate the necessity for balance of quality, quantity, and cost in home construction
4. To learn to do some of the computations required in home building and maintenance
5. To learn to estimate and use approximate numbers

Content	Procedures and Activities	Teaching Aids
A. The Lot 1. Selection of neighborhood	Invite local real estate agent to speak on the factors governing lot evaluation	
2. Use of surveying tools	Invite surveyor to demonstrate the use of instruments	Shuster, C. N., and Bedford, F. L. *Field Work in Mathematics*. Yoder Instrument Co., 1935 Boy Scouts of America. Merit Badge Booklet #3327 *Surveying* If available, show film: *Fundamentals of Land Surveying*. University of California Regional Source: Selected Films, Inc., Film Rental Dept., 1018 So. Wabash Ave., Chicago 5, Ill.
a. Transit and poles	Make a transit or a hypsometer	

	Make a traverse on the school grounds. This could take the form of a treasure hunt	
	Make an ancient surveying instrument	
(1) Lot division and lot description	Invite local building inspector to tell about restrictions and laws	Federal Housing Administration, Washington, D. C. "Principles of Planning Small Houses," Technical Bulletin No. 4 pp. 40-44
	Make a large wall map of a subdivision with different shaped lots, give it a name and letter students' names on purchased lots. Later, they can cut a silhouette of their house plan and paste it on the lot in the proper position. One of the students could act as the agent selling the lots	
	Study principles of scale drawing and make a large area map	
	Have students draw individual plats of a subdivision and buy and record lots	If available, show film: *Caught Mapping*. General Motors Corp., Film Library, Detroit 2, Mich. Free
	Map an area using alidade and plane table	
(2) Problems of leveling	Calculate water and sewer lines in subdivision	If available, show film: *Level Protractor*. United World Films, 1445 Park Ave., New York 29, N. Y.
	Make a level	
	Plot elevations on lots	
B. The House Plan		
1. Reading blueprints and house plans	Collect plans from local lumber dealers and magazines	
	Review and emphasize the principles and uses of ratio and proportion	
2. Use of scales in drawing house plans	Draw house plans of own design to be built on purchased lots	Federal Housing Administration, Washington, D. C. "Principles of Planning Small Houses," Technical Bulletin No. 4
		If available, show film: *Principles of Scale Drawing*. Coronet Films, Coronet Bldg., Chicago 1, Ill.

	Discuss problems of alignment of house on lot	
	Check area and placement of house to conform to local building restrictions	If available, show film: *Areas.* Knowledge Builders, Visual Educ. Bldg., Floral Park, N. Y.
	Calculate areas of collected house plans	
		Townsend, Gilbert, Dalzell, J., and McKinney, J. *How to Estimate For the Building Trades.* American Technical Society, 1955 or similar source
3. Estimation of cost of house	Calculate house costs by quoted prices per square foot	
	Consider budget allowance in determining size of home to be built	
C. The Construction of the House		If available, show film: *Homes Unlimited.* Free. Modern Talking Pict. Service, 3 E. 45th Street, New York 19, N. Y.
1. Costs of materials	Invite local lumber dealers to discuss costs and types of lumber	
	Collect information and samples from dealers, magazines, and newspapers on costs and types of materials	
	Make scrapbook of items desired in own home	
	Estimate costs by learning to compute with approximate numbers	If available, show film: *Measurement.* Coronet Films, Coronet Bldg., Chicago 1, Ill.
		Van Leuvan, Edwin P. *General Trade Mathematics.* McGraw, 1952 or Other comparable trade mathematics textbook
		History of Measurement Charts. Free. Ford Motor Company, Educational Relations Dept., Dearborn, Michigan
		Johnson, D. A., and Glenn, W. H. *The World of Measurement.* Webster, 1961
		The Amazing Story of Measurement. Lufkin Co., Saginaw, Michigan

	Have students form own companies to bid on and supply materials for individual house constructions	
	Keep record of costs for each house	
a. Excavating	Use volume formulas	
b. Cement work	Calculate costs of block work, steps, floors, walks, for example	
c. Lumber	Calculate board feet and cost of lumber	
d. Roofing materials	Calculate shingle and other types of roofing costs	
e. Glass, tile, brick	Calculate number of each needed	
	Investigate tessellations by use of tile samples	
f. Paint and paper	Calculate areas and amounts required	
	Calculations of the above types could easily be overdone. Watch for a point of diminishing returns and lack of interest	
2. Construction details	Investigate the use of geometric principles by means of the tools of the carpenter, particularly the carpenter's square	Johnson, D. A. and Glenn, W. H. *The Pythagorean Theorem*. Webster, 1961
	Make a model of a house or some of the construction details	
	Make a field trip to a house under construction	
3. Cost of home	Calculate total cost	
	Investigate various trades and building vocations to learn about training needed, mathematics used, wages, and other factors. Students may be able to give more complete reports on assumed job during project	
	Discuss installment payments. Relate to previous study on installment buying in Unit VII	

D. Landscaping	
1. Cost of preparing soil	Calculate sod, seed, shrubbery costs
2. Cost of upkeep	Calculate water, fertilizer, mowing costs
E. Maintenance of Home	
1. Gas, water, electricity	Learn to read meter and calculate cost
2. Insurance	Discuss various types and costs. Relate to previous study of insurance
3. Heating	Discuss various types and costs
4. Property taxes	Find how the property tax rate is calculated. Refer to previous study of taxes
	Learn what is meant by "special" assessments and "homestead" exemption
5. Total maintenance cost per year	Discuss home ownership vs. renting
	Determine upkeep costs such as: painting, repapering, repairing
F. Furnishing the home	
1. Carpeting	Calculate areas and costs
2. Appliances	Discuss buying "on time" vs. cash. Relate to previous study of installment buying
3. Home furnishings	Make a collection of pictures illustrating the use of geometric design in home furnishings
4. Draperies	Calculate drapery material needed for various pleating depths. This may be a "girls only" project
G. Improvement of home	
1. Do-it-yourself projects	Encourage students to report on projects undertaken in their own homes emphasizing the mathematics involved

Have students present individual reports on mathematics used, materials needed, and costs of projects, such as fencing, car port, garage, swimming pool, retaining walls, recreation room panelling

BLAST OFF to the FUTURE

UNIT X

BLAST OFF TO THE FUTURE

Introduction

Space flight is here to stay! Man's insatiable curiosity, coupled with his insecure relation with people of other national origins compels him outward toward the stars. This is an era of astronauts and sputniks, and we must learn about them to live in our society.

Needless to say, space flight requires a vast knowledge of mathematics. In this unit, the mathematics needed for a basic understanding of space travel is introduced. It starts with the simplest of coordinate geometries, the geometry of the line. From this it progresses to two-dimensional and three-dimensional coordinate systems, starting with basic concepts and proceeding inductively.

Inductive thinking is the key to the presentation of this unit. Proofs are not given nor expected.

The unit also provides an introduction to the conic sections, parabolas, ellipses, and hyperbolas, an understanding of which is a necessary prerequisite to the comprehension of space flight.

Signed numbers and absolute value are reintroduced and an acquaintance with the Pythagorean Theorem is presupposed. There are many teaching aids and references available, and these needs should be anticipated before starting the unit.

The teacher should feel free to use any part of this unit which will be acceptable to the particular class he is teaching. He should make every effort to have the students discover the relationships presented rather than lecture on each topic. The usual short, frequent, and informal drill periods should be continued while studying this unit.

Objectives

1. To impress upon the student the importance of mathematics in the struggle for the conquest of space
2. To develop an understanding of some elementary mathematical concepts of space travel
3. To broaden the students understanding of the concepts of algebra and geometry within the context of the space age
4. To provide the students with interesting by-ways and areas of exploration related to distances, direction, and dimension

Content	Procedures and Activities	Teaching Aids
A. Coordinate Geometry 1. Location of points on a line	Since a line has no beginning and no end, the students can select an arbitrary zero point. Have the students locate points corresponding to the positive integers first. Then locate the positive fractions and decimal fractions	In addition to the references found in each section of the outline, there are several important sources of references and aids which the teacher should consult in planning the work of the unit. Berger and Johnson, "A Guide to the Procurement of Teaching Aids for Mathematics," National Council of Teachers of Mathematics, April, 1959. William L. Schaaf, "Recreational Mathematics," NCTM, 1958. Teachers should also refer to the yearly indices printed in the December issues of *The Mathematics Teacher*

a. Setting up a one-to-one correspondence between numbers and points on the line

b. One and only one coordinate is necessary to locate the position of any point on a line

At this point the teacher might like to digress a bit and have the pupils explore the concept of "denseness." How many points are found between 1 and 2? Between .1 and .2? Between .01 and .02? This discussion could add to an understanding of one-to-one correspondence

Pupils should now be exposed to the location of negative numbers. It might be desirable to use the time-honored thermometer as an illustration but most of these pupils have some knowledge of the existence of such numbers

2. Determining distances between point on a number line

   a. By counting units between the two locations

   b. Develop a distance formula for the number line to the generalization $(X_2 - X_1) = d$

Here the pupils may be asked to determine the distance between many sets of points by first locating the points, then counting the units between them. A chart such as the one below will help them to discover the pattern and thus the generalization. It is important to begin with the positive integers. This will enable the students to see the pattern develop more clearly

X_1	X_2	Distance
5	2	
10	1	
8	6	
7	7	
2	5	
1	9	
½	2½	
3¼	1¾	
− 4	− 2	
−7	3	
0	− 5	
− ½	2¼	
6.25	− 4.75	

Pupils should do as many of these as necessary to see the pattern. When the generalization has been formed, pupils should determine distances

between points without graphing the points and counting

Here is a place where the student can be expected to develop the formula by himself, and display the enthusiasm which accompanies this discovery

3. Location of points on a plane

   a. Two and only two coordinates are necessary to locate or to specify any point in a plane

   b. An ordering of the two coordinates is necessary to specify a particular point

   c. Develop a distance formula for any two points in a plane

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The concept of using two coordinates to specify a point can be introduced nicely with a map using lines of longitude and latitude for positioning cities. After some discussion of this technique of mapping, most students should be ready to abstract the lines of latitude and longitude to the Cartesian grid. If the need for an ordering of the coordinates has not yet been raised by the pupils, the points such as (2,5) and (5,2) should be plotted and the students should be asked for suggestions which would specify the exact point to be discussed. The teacher might like to return to the map and locate two cities whose latitude and longitude are reversed

Mathematics for the Junior High School Volume II. Yale University Press, 1961. Chapter I

By this time, pupils should recognize the two conditions required for specifying any point on a plane; namely, two coordinates and an ordering of the coordinates. The notation for an ordered pair of numbers should be introduced and the pupils should receive extensive practice in plotting points. This practice can be made more intriguing to pupils if the coordinates, when connected, trace out interesting pictures

In order to develop the distance formula, it will be necessary to review with the students the Pythagorean theorem. All of the pupils will have had some exposure to the principle before this course is taken but the extent to which the concept must be reviewed will depend upon the nature of the class. A good beginning would be to place two points on the grid, one of the x axis and one on the y axis; (0,3) and (4,0). To measure the distance between the points the students might suggest spreading the compass

Jackson, R. L. "The Discovery of a Concept—A Demonstration Lesson." *Mathematics Teacher*. March, 1961

between the points, then laying off the distance on the x axis and counting the squares. In this case the method would be appropriate provided the measurement is accurate. Of course the technique breaks down when the distance becomes an irrational number. This problem can be pointed out by selecting two other coordinates. Thus the need for a technique (the Pythagorean theorem in this case) is created

The Pythagorean relationship can easily be translated to the distance formula by using the concept developed in IB2, using the above coordinates:

$$d = \sqrt{X^2 + Y^2}$$

Since $X = (3 - 0)$ and $Y = (4 - 0)$

$$d = \sqrt{(3 - 0)^2 + (4 - 0)^2}$$

Other coordinates can now be used. Pupils must be led slowly to this concept with constant reference to the Cartesian coordinate grid

In selecting problems for the pupils, points should be selected only in quadrant I first, followed by points selected from other quadrants. It may be necessary to review with the students at least an intuitive idea of absolute value

4. Location of points in space

a. Three and only three coordinates are necessary to locate or to specify any point in three dimensional space	Pupils will experience some difficulty in visualizing three dimensional space on a two dimensional surface. However from the preceding development, many will be able to predict the number of coordinates needed to locate a point in space and also to recognize the need for ordered triplets of numbers	Something like D-Stix sets, available commercially, may be of help here
b. An ordering of the three coordinates is necessary to specify a particular point	It might be advantageous to spend a brief period in perspective drawing. No doubt most teachers have a variety of techniques which aid in overcoming this problem	National Council of Teachers of Mathematics, *Multi-Sensory Aids in the Teaching of Mathematics*. 18th yearbook, 1945

c. Develop a distance formula for any two points in space

Some three dimensional graphing and plotting of points should occur at this time. Pupils may be interested in constructing models to illustrate the location of points in space. Some curve stitching activities can add real interest at this point. Pupils may wish to construct three dimensional string models out of fluorescent elastic thread. Perhaps a kit such as "Space Geometries" would be helpful here

As the pupils contrast the formulas for distances on one and two dimensional surfaces, several may come up with the generalization for three dimensional distances. ALWAYS MAKE USE OF STUDENT HUNCHES

5. Explore the possibilities of a fourth dimension

Generalizations of the above concepts may be made to a fourth dimension. Some students may be interested in exploring this possibility; even a construction of a fourth dimensional model (the tesseract). The extent to which a teacher can dwell on this topic must be determined by the interest and ability of the class

Abbott's book, "Flatland" offers an interesting setting for a lively discussion of life as it might be lived in zero, one, two, and four dimensions. STUDENT ACTIVITY AND INVOLVEMENT IS THE KEY SUCCESS WITH THIS MATERIAL

Abbott, Edwin A. *Flatland: A Romance of Many Dimensions.* Dover, 1952

Gamow, George. *One Two Three—infinity.* New Am. Library, 1954

Kline, Morris. *Mathematics in Western Culture.* Oxford Univ. Press, 1953

B. Velocity, acceleration, and gravitation

1. The meaning of speed and velocity
   a. Speed indicates how fast an object is moving

   b. Velocity indicates the rate of motion in a particular direction

2. The meaning of acceleration
   a. Acceleration is the rate of change of velocity

Since much of the work in this unit is dependent upon an understanding of the concept of velocity and acceleration, particular pains should be taken in presenting these concepts. Most high school physics books provide a number of exercises and illustrations dealing with velocity and acceleration. These exercises should include distance, velocity, and time relationships using the expression $d = v\,t$

Explore acceleration of a car. Have some students gather data including time and speed at differing acceleration rates. Graphing speed against time will show acceleration. This would be a good class project

Mills, Robert. "The Man in Motion." *The Mathematics Teacher.* Feb., 1961. pp 107-109

Rush, J. H. "The Speed of Light." *Scientific American.* Aug., 1955. p. 62

Weidemann, Charles C. "Animal and Bird Speeds." The Mathematics Teacher. Feb., 1951. pp 142-143

Larson, Harold D. *Faster and Faster.* Row Peterson, 1956

Meyer, Sheldon. *The Mathematics Teacher.*

3. Newton's first and second laws of motion a. An object in motion will not change its speed or direction unless acted upon by a force b. Force is equal to the product of the mass and its acceleration	Use a gyroscope to indicate inertia	Application Section, October, 1952. pp 453-454
4. Acceleration due to gravity	This concept should be mentioned briefly in passing	
C. Curves of falling objects—the parabola 1. Dropping an object a. $d = 16\ t^2$ b. The effect of the atmosphere on a freely falling object	The coin-feather tube commonly found in physics laboratories offers another interesting demonstration. The result of this experiment might lead the class into an interesting discussion of gliders and the effect of the at nosphere on their flight	
2. Dropping relief packages or bombs from a moving object a. Two forces are involved in this situation; that of gravity acting on the object and that of the moving object itself	This is the first application in this unit of Newton's first law of motion. A graphical or pictorial presentation of the falling object would probably be the most effective method of illustration. This picture leads nicely into the concept of a parabolic curve. Some discussion will no doubt develop regarding the problem of how a bombardier can determine how far ahead of the target the bomb should be released. Bomb sights are often available from war surplus stores and can add interest to class discussion	Kline, Morris. *Mathematics and the Physical World.* Crowell, 1959 Bakst, Aaron. *Mathematics, Its Magic and Mastery.* Van Nostrand, 1952
b. The curve of descent of such an object is parabolic in nature c. $d = vt - 16t^2$		
3. Javelins, rifles, artillery, and ICBM's a. The motion of objects to be propelled horizontally and upward is dependent upon the vertical and horizontal velocity and the vertical and horizontal distance to be covered	The student should be aware of the fact that this trajectory is a curve similar to that discussed in the section on dropping relief packages and bombs. The discussion of the trajectory of a shell should bring out the importance of the angle of elevation of the launcher in determining the distance to be traveled by the missile	

$y = (u/v)\,x + 16\,x^2 / u^2$
where
x = horizontal distance
y = vertical distance
u = horizontal velocity
v = vertical velocity

A discussion of this concept can be found in Aaron Bakst's book, "Mathematics, Its Magic and Mastery," Chapter 36, entitled, "The Firing Squad and Mathematics"

If the class is largely sports oriented, the same concept can be developed from various athletic events such as the javelin throw in track, kicking, and passing in football, batting in baseball and shooting in basketball

Students can take time exposures of someone shooting a basketball with differing arcs. With a strong light, the reflection will trace a parabola

4. The nature of the parabola

   a. A parabola is the locus of a point which moves in a plane so that its distance from a fixed point is always equal to its distance from a fixed line

A number of graphing exercises involving the parabola offer the student an opportunity to investigate the relationships between changes in the general formula and the shape of the curve. Examples might be the addition of coefficients other than 1 for each of the variables, introducing negative coefficients, and interchanging the variables of the equation

   b. The axis, directrix and focus of the parabola

   c. The equation of the parabola: $ay^2 = bx + c$

   d. Graphing the parabola

5. The construction of the parabola

   a. Paper folding

   b. The parabola as a conic section

Several interesting laboratory sessions can give the pupils a good understanding of methods of constructing the parabola. Paper folding exercises can be found in the pamphlet, *Paper Folding in the Mathematics Classroom* by D. A. Johnson.

   c. The use of a compass and straight edge

   d. Curve stitching

Curve stitching exercises are explained and discussed in the pamphlet, *A Rhythmic Approach to Mathematics* by Edith Somervell. Kits are available commercially which contain materials for constructing many interesting geometric patterns with elastic thread

Struyk, Adrian. "Theme Paper, a Ruler and the Central Conics." *The Mathematics Teacher*. March, 1954. pp. 189-193

Olmsted, Hugh. "A Parabola Device." *The Mathematics Teacher*. Oct., 1955. pp. 407-408

Johnson, D. A. *Paper Folding in the Mathematics Classroom*. National Council of Teachers of Mathematics, Washington, D. C.

Somervell, Edith. *A Rhythmic Approach to Mathematics*

6. Uses of the parabolic curve a. The development of the paraboloid by the rotation of the parabola on its axis b. In reflection of light c. In radio reception and transmission d. As a nomograph in computing squares and square roots	Uses and occurrences of the parabola can be found in the pamphlet called *Curves in Space* by D. A. Johnson. This section of the unit can also stimulate a good deal of class discussion. Flashlight beams, automobile headlights, reflecting telescopes, for example, offer many possibilities for these discussions	Johnson, D. A. *Curves in Space.* Webster, 1963 Saupe, Ethel. "Simple Paper Models of the Conic Sections." *The Mathematics Teacher.* Jan., 1955. pp. 42-44 See Unit I on Computing Machines
7. Overcoming the gravitational attraction of the earth a. The escape velocity of the earth is approximately 7 miles per second b. Human tolerance of high acceleration rates c. Anti-gravity machines	Some of the historical background of rocket work which would be interesting to the students is found in Willy Ley's book on *Rockets, Missiles and Space Travel.* The construction of rockets, the development of fuels and recent innovations to aid in the development of needed thrust are discussed in simple language and in great detail	Ley, Willy. *Rockets, Missiles and Space Travel.* Viking, 1958 Teller, Edward. "The Geometry of Space and Time." *The Mathematics Teacher.* Nov., 1961. pp. 505-514
D. The Curve of Space Travel—the ellipse 1. Understanding the motions of the earth a. The earth rotates on its axis at a speed of approximately 1,000 mph at the equator b. The earth's revolution about the sun: 65,000 mph c. The earth's movement through the galaxy: 60,000 mph d. The galaxy's movement through the universe is of unknown speed (3,500,000 mph—estimate)	An understanding of the complex motions of the earth and its neighbors will help the students see the complexities of launching space vehicles and having them arrive in space at the right place and the right time. This difficulty will be explored further in section E	

2. The nature of the ellipse

   a. An ellipse is the locus of a point which moves so that the sum of its distances from two fixed points is constant

      (1) The two fixed points are called foci

      (2) The ellipse has two axes

   b. The equation of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

   c. Graphing the ellipse

3. The construction of the ellipse

   a. By the use of string and pins

   b. Construction with compasses and straight edge — See section B-5 on the parabola

   c. Paper folding

   d. Curve stitching

   e. The ellipse as a conic section

4. Uses of the elliptical curve — The laboratory approach can be quite successful in this section. The pupils might like to construct items like an elliptical billiard table or swimming pool which illustrates beautifully the concentration of waves and shots emanating from one focus and converging on the other focus

   a. The generation of an ellipsoid by rotation of the ellipse on either the major or minor axis

   b. Whispering galleries

   c. Reflection of sound, light and waves

   d. Gears — Students may wish to experiment with elliptical gears and perhaps follow through with some study of curves of constant diameter

Hazard, William. "Curves of Constant Breadth." *The Mathematics Teacher.* Dec., 1955. pp. 89-90

5. Orbiting of planets and satellites

   a. Kepler's great discovery—the elliptical orbit of the planets

      $\frac{T^2}{D^3} = k$

      Where T is the period of revolution of the body, D is the average distance from the sun and K is a constant for all bodies

   b. Specific factors involved in the orbiting of satellites

      (1) Preventing the satellites from escaping the earth's gravitational force completely

      (2) Preventing the satellites from returning to the earth's atmosphere and burning up

      (3) Minaturization of instruments allowing small pay loads to gather important data

The historical development of Kepler's works can prove to be very helpful here. Morris Kline's book, *Mathematics and the Physical World*, and Bakst's book, *Mathematics, Its Magic and Mastery* would be most helpful

Whipple, F. L., and Hynek, J. A. "Observations of Satellite I." *Scientific American*. Dec., 1957. p. 37

Mengel, J. T., and Herget, Paul. "Tracking Satellites by Radio," *Scientific American*. Jan., 1958. p. 23

Landon, M. V. "Mechanics of Orbiting," *The Mathematics Teacher*. May, 1959. pp. 361-364

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E. Space landings and interplanetary travel

   1. The complexity of earth motions and target motions in scheduling meetings of the spacecraft and the target

      a. Lunar probes

This section will help the pupil understand why there is more involved in "hitting the moon" than merely pointing a rocket in that direction. Adding the motion of the moon to the complex movements of the earth makes the problem even more involved. Similar problems will exist when attempting a rendezvous with a planet. Since lunar probes are presently underway, they would serve as a good starting point for this discussion

   2. Possibilities of interplanetary travel and travel outside our solar system

Using as a basis the optimum speed possible at this time in outer space travel, have the pupils compute the length of time needed to make the

a. The time factor	journey to other planets as well as to the nearest star. The pupils will be interested in discussing the maximum possible speed of our future space ships	
	Publications prepared by NASA (National Aeronautics and Space Administration) are most helpful in providing up to date information on these problems. A recent book, *Projects: Space*, by Judith Viorst was written specifically on NASA projects	Viorst, Judith. *Projects: Space.* Washington Square Press Inc., 1962
F. The curve of location: the hyperbola	Stress the double figure inherent in the graph of the hyperbola	See references on the parabola, section C-5
1. The nature of the hyperbola		
a. its graph and equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$		
b. asymptotes		
2. Uses of the hyperbola		
a. The paths of some comets travel in hyperbolic curves	Mention that the hyperbola is a starting point for the analytic study of logarithms. See any modern advanced algebra text for a discussion	
b. Uses in navigation in determining locations on the earth's surface		
c. The sonic boom		
3. Construction of the hyperbolic curve		Struyk, Adrian. "Theme Paper, a Ruler and the Hyperbola." *The Mathematics Teacher.* Jan., 1954. pp. 29-30
a. Using the compass and straight edge		
b. Paper folding		
c. The hyperbola as a conic section		

G. Other topics to investigate

1. Other curves and their applications such as the catenary, cardioid, cycloid, spiral	Mention that the catenary is the "curve of quickest descent." See a text on Calculus of Variations.	
2. Other curved surfaces such as the torus, moebius strip, and pretzel	Ask the class if an inner tube (torus) can be turned inside out	
3. Linkages as mechanical devices for constructing curves		N.C.T.M., 18th yearbook 1945. "Multi-Sensory Aids."
4. Vectors		

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